Optimal Packings of Lines.

Prof. Dr. Matthew Fickus
AFIT
Matthew.Fickus@afit.edu

What's the best way to arrange a given number of lines in a finite-dimensional Euclidean space? In particular, when the number of lines exceeds the dimension of the space, how should we design these lines so that they are as orthonormal as possible? This question arises in many real-world applications involving communications and compressed sensing. As we shall discuss, its answer depends a lot on our choice of metric for near-orthonormality. Minimizing one popular metric known as "coherence" yields lines with the property that the minimum angle between any two lines is as large as possible. With regard to this topic, we discuss some new methods for constructing minimally-coherent collections of lines known as equiangular tight frames (ETFs), and consider the degree to which they satisfy compressed sensing's restricted isometry property. However, there are situations where no ETF exists, and in those cases, we understand little about minimizing coherence. There, we turn to a simpler metric for near-orthonormality known as the "frame potential" whose minimizers are unit-norm tight frames (UNTFs). We discuss a few recent advances in the study of UNTFs, and how they might be used to better understand ETFs and other minimally-coherent collections of lines.