Summer School on Numerical Linear Algebra for Dynamical and High-Dimensional Problems
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Solution of Computational Problems for Descriptor Systems

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Descriptor Systems

Given: LTI descriptor system

$$\Sigma: \quad \begin{cases} E\dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t) + Du(t), \end{cases}$$

- $E, A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$, $D \in \mathbb{R}^{p \times m}$,
- descriptor vector $x(t) \in \mathbb{R}^n$, input vector $u(t) \in \mathbb{R}^m$, output vector $y(t) \in \mathbb{R}^p$.
- Assumptions: $\lambda E - A$ is regular, i.e. $\det(\lambda E - A) \not\equiv 0$.

Transfer function

$$G(s) = C(sE - A)^{-1}B + D$$

Additional Structures

Bounded realness

- G has no poles with nonnegative real parts,
- $I - G(\mathrm{i}\omega)G^H(\mathrm{i}\omega) \succcurlyeq 0$ for all values $\omega \in \mathbb{R}$.

Positive realness

- G has no poles with positive real parts,
- $G(\mathrm{i}\omega) + G^H(\mathrm{i}\omega) \succcurlyeq 0$ for any $\mathrm{i}\omega$ that is not a pole of G with $\omega \in \mathbb{R}$,
- if $\mathrm{i}\omega$ or ∞ is a pole of G, then it is simple and the relevant residue matrix is positive semidefinite Hermitian.

General frequency domain inequalities (FDIs)

$$\begin{bmatrix} (\mathrm{i}\omega E - A)^{-1}B \\ I \end{bmatrix}^H \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} (\mathrm{i}\omega E - A)^{-1}B \\ I \end{bmatrix} \succcurlyeq 0, \; Q = Q^T, \; R = R^T.$$

Connection to Linear Matrix Inequalities

Bounded real lemma

Under certain conditions: G is bounded real if and only if the LMI

$$\begin{bmatrix} A^T X + X^T A + C^T C & X^T B + C^T D \\ B^T X + D^T C & D^T D - I \end{bmatrix} \preccurlyeq 0, \; E^T X = X^T E \succcurlyeq 0 \text{ is feasible.}$$

Positive real lemma

Under certain conditions: G is positive real if and only if the LMI

$$\begin{bmatrix} A^T X + X^T A & X^T B - C^T \\ B^T X - C & -D^T - D \end{bmatrix} \preccurlyeq 0, \; E^T X = X^T E \succcurlyeq 0 \text{ is feasible.}$$

Kalman-Yakubovič-Popov lemma

Under certain conditions: FDI holds for all $\mathrm{i}\omega$ if and only if the LMI

$$\begin{bmatrix} A^T X + X^T A + Q & X^T B + S \\ B^T X + S^T & R \end{bmatrix} \succcurlyeq 0, \; E^T X = X^T E \text{ is feasible.}$$

Problems under Consideration

Weakening equivalence conditions

The conditions which are needed to state equivalence are quite strong at the moment $\Longrightarrow$ weaking this conditions to make it more practical.

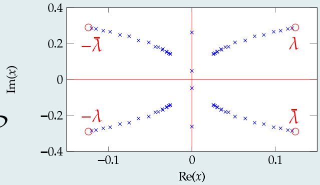
Passivity enforcement

Bounded realness/positive realness/FDI are natural properties of real-world systems. Often these are lost due to modeling errors/approximation errors $\Longrightarrow$ restore this by introducing small errors to system's matrices (can be done by perturbation of purely imaginary eigenvalues of skew-Hamiltonian/Hamiltonian matrix pencils (next slide)).

Skew-Hamiltonian/Hamiltonian Matrix Pencils

Properties

- block structure: $\lambda\mathcal{S} - \mathcal{H} = \lambda \begin{bmatrix} F & G \\ H & F^T \end{bmatrix} - \begin{bmatrix} R & S \\ T & -R^T \end{bmatrix}$ with skew-Hermitian G, H, and Hermitian S, T,
- Hamiltonian eigensymmetry (symmetry with respect to imaginary axis (and the real axis in the real case)),



- pencils $\lambda\tilde{\mathcal{S}} - \tilde{\mathcal{H}} := \mathcal{J}\mathcal{P}^T\mathcal{J}^T(\lambda\mathcal{S} - \mathcal{H})\mathcal{P}$ with $\mathcal{J} = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$ are again sH/H,
- structured Schur form:

$$\mathcal{J}\mathcal{Q}^T\mathcal{J}^T(\lambda\mathcal{S} - \mathcal{H})\mathcal{Q} = \lambda \begin{bmatrix} S_1 & S_2 \\ 0 & S_1^T \end{bmatrix} - \begin{bmatrix} H_1 & H_2 \\ 0 & -H_1^T \end{bmatrix},$$

with orthogonal $\mathcal{Q}$, upper triangular S_1, and H_1; however existence cannot be guaranteed, can be solved by structured embedding.

Problems under Consideration

Structure-preserving balancing

Goal: find a simple $\mathcal{J}$-congruence transform such that:

- in the transformed pencil a maximum number of eigenvalues has been isolated (i.e., part of the pencil is in structured Schur form),
- in the remainder, the rows and columns of the pencil are as close in norm as possible (to improve numerical accuracy).

Perturbation theory for purely imaginary eigenvalues

For many applications the interesting eigenvalues of skew-Hamiltonian/Hamiltonian pencils are the purely imaginary ones.
Question: How to move these eigenvalues off the imaginary axis by structured perturbations?

- might be possible by arbitrarily small perturbations if some of the purely imaginary eigenvalues are not simple,
- depends on the skew-Hamiltonian/Hamiltonian Kronecker canonical form.