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"Dynamics and Control Theory"
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Passivity Enforcement of Descriptor Systems via Structured Perturbation of Hamiltonian Matrix Pencils

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(joint work with Peter Benner)

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Motivation

Given: Continuous-time LTI descriptor system

$$\Sigma : \quad \begin{cases} E\dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t) + Du(t), \end{cases}$$

- $E, A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{m \times n}$, $D \in \mathbb{R}^{m \times m}$,
- Descriptor vector $x(t) \in \mathbb{R}^n$, input vector $u(t) \in \mathbb{R}^m$, output vector $y(t) \in \mathbb{R}^m$.
- Assumptions: $\lambda E - A$ is regular and stable, $(E; A, B, C, D)$ is C-controllable and C-observable (i.e., $\operatorname{rank}\left(\begin{bmatrix} \alpha E - \beta A & B \end{bmatrix}\right) = n$ and $\operatorname{rank}\left(\begin{bmatrix} \alpha E^T - \beta A^T & C^T \end{bmatrix}\right) = n$ for all $(\alpha, \beta) \in \mathbb{C}^2 \setminus \{(0,0)\}$).

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Transfer function

$$G(s) := C(sE - A)^{-1}B + D$$

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Goal

Find a passive system

$$\tilde{\Sigma}: \quad \begin{cases} \tilde{E}\dot{x}(t) = \tilde{A}x(t) + \tilde{B}u(t), \\ y(t) = \tilde{C}x(t) + \tilde{D}u(t), \end{cases}$$

with transfer function $\tilde{G}$ such that $\|\tilde{G} - G\|$ is small in some system norm!

Passivity Enforcement in the Literature

- [Grivet-Talocia '04]: passivity enforcement of standard state-space systems via perturbation of Hamiltonian matrices,
- [Schröder, Stykel '07]: using structure-preserving algorithms to compute required eigenvalues and eigenvectors,
- [Wang, Zheng, Koh, Pang, Wong '10]: passivity enforcement of descriptor systems using skew-Hamiltonian/Hamiltonian matrix pencils, explicit computation of spectral projectors to extract relevant subsystems,

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- [Wang, Zheng, Koh, Pang, Wong '10]: passivity enforcement of descriptor systems using skew-Hamiltonian/Hamiltonian matrix pencils, explicit computation of spectral projectors to extract relevant subsystems,
- Here: structure-preserving computation of required eigenvalues and eigenvectors, no projectors needed.

1. Preliminaries
2. Computation of the System Decomposition
3. Passivity Enforcement of the Proper Part
4. Numerical Example
5. Conclusion and Outlook

1 Preliminaries

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Passivity Concepts

"Passivity is the inability of a system to generate energy."

Passivity Concepts

Scattering representation

A descriptor system Σ in scattering representation is called (strictly) passive if

$$\int_0^t \|u(\tau)\|^2 - \|y(\tau)\|^2 \mathrm{d}\tau \geq 0 (> 0)$$

for all $t > 0$, all $u \in \mathcal{L}_2([0, t], \mathbb{R}^m)$ and consistent initial conditions. This property is also often called contractivity.

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Admittance/impedance representation

A descriptor system Σ in admittance/impedance representation is called (strictly) passive if

$$\int_0^t u(\tau)^T y(\tau) \mathrm{d}\tau \geq 0 (> 0)$$

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Bounded Realness

Bounded real transfer function

The transfer function G is called (strictly) bounded real, if

- G has no poles with nonnegative real parts,
- $H(\mathrm{i}\omega) := I - G(\mathrm{i}\omega)G^H(\mathrm{i}\omega)$ is positive semidefinite (positive definite) for all values $\omega \in \mathbb{R}$.

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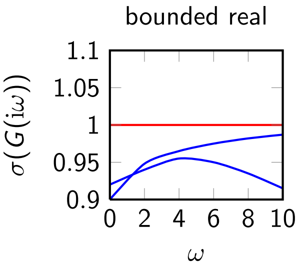

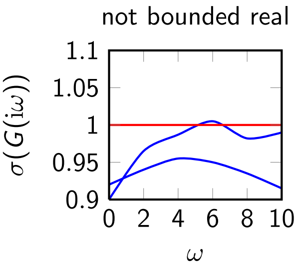

Positive Realness

Positive real transfer function

The transfer function G is called (strictly) positive real, if

- G has no poles with positive (nonnegative) real parts,
- $H(\mathrm{i}\omega) := \frac{1}{2}\left(G(\mathrm{i}\omega) + G^H(\mathrm{i}\omega)\right)$ is positive semidefinite (positive definite) for any $\mathrm{i}\omega$ that is not a pole of G with $\omega \in \mathbb{R}$,
- $\mathrm{i}\omega$ or ∞ is a pole of G, then it is simple and the relevant residue matrix is positive semidefinite Hermitian.

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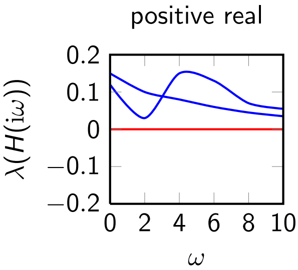

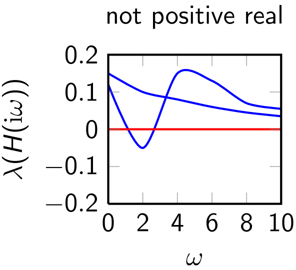

System Decomposition

Laurent series expansion of $G(s)$ at $s = \infty$

$$G(s) = \underbrace{\underbrace{\sum_{k=-\infty}^{-1} M_k s^k}_{=:G_{sp}(s)} + M_0}_{=:G_p(s)} + \underbrace{\sum_{k=1}^{d} M_k s^k}_{=:G_i(s)}$$

- G_{sp}: strictly proper part, i.e., $\lim_{\omega \to \infty} \|G_{sp}(\mathrm{i}\omega)\| = 0$,
- G_p: proper part, i.e., $\lim_{\omega \to \infty} \|G_p(\mathrm{i}\omega)\| < \infty$,
- G_i: improper part, i.e. $\lim_{\omega \to \infty} \|G_i(\mathrm{i}\omega)\| = \infty$.

Equivalent Passivity Conditions

Scattering representation

A descriptor system Σ in scattering representation is (strictly) passive if and only if

- the proper part $G_p := G_{sp} + M_0$ is (strictly) bounded real,
- $M_k = 0$ for $k \geq 1$.

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Admittance/impedance representation

A descriptor system Σ in admittance/impendance representation is (strictly) passive if and only if

- the proper part $G_p := G_{sp} + M_0$ is (strictly) positive real,
- M_1 is symmetric positive semidefinite ($M_1 = 0$).
- $M_k = 0$ for $k \geq 2$.

General Outline of Passivity Enforcement Procedure

Basic Steps

1. Decouple G into its proper and improper part.
2. Check, if $M_k = 0$ for $k \geq 1$ (scattering representation) or $k \geq 2$ (admittance/impedance representation) $\Rightarrow$ if not, passivity cannot be enforced.
3. Enforce bounded/positive realness of the proper part.
4. Enforce positive semidefiniteness of M_1 for systems in admittance/impedance representation.

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Reduction of the System Pencil

Theorem [Benner, Chu '05]

For any regular matrix pencil $A - \lambda E$ there exist orthogonal matrices $U, V \in \mathbb{R}^{n \times n}$ such that

$$U(A-\lambda E)V = \begin{array}{c} \begin{array}{cccc} n_1 & n_2 & n_3 & n_4 \end{array} \\ \left[\begin{array}{cccc} A_{11}-\lambda E_{11} & A_{12}-\lambda E_{12} & A_{13}-\lambda E_{13} & A_{14}-\lambda E_{14} \\ 0 & A_{22} & A_{23}-\lambda E_{23} & A_{24}-\lambda E_{24} \\ 0 & 0 & A_{33} & A_{34} \\ 0 & 0 & 0 & A_{44} \end{array}\right] \begin{array}{l} \}n_1 \\ \}n_3 \\ \}n_2 \\ \}n_4 \end{array} \end{array},$$

where $\operatorname{rank}(E_{11}) = n_1$, $\operatorname{rank}(E_{23}) = n_3$, $\operatorname{rank}(A_{44}) = n_4$, and

$$\operatorname{rank}\left(\begin{bmatrix} A_{22} & A_{23}-\lambda E_{23} \\ 0 & A_{33} \end{bmatrix}\right) = n_2 + n_3 \quad \forall \lambda \in \mathbb{C}.$$

Distinction of Cases [Benner, Chu '05]

Case 1: $n_2 \neq n_3$

- There exist $M_k \neq 0$ with $k \geq 2$.
- System Σ is non-passive and passivity cannot be enforced.

Distinction of Cases [Benner, Chu '05]

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- There exist $M_k \neq 0$ with $k \geq 2$.
- System Σ is non-passive and passivity cannot be enforced.

Case 2: $n_2 = n_3 = 0$

- We have $M_k = 0$ for $k \geq 1$ and

$$\begin{aligned} G(s) &= \begin{bmatrix} C_1 & C_4 \end{bmatrix} \left(s \begin{bmatrix} E_{11} & E_{14} \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} A_{11} & A_{14} \\ 0 & A_{44} \end{bmatrix} \right)^{-1} \begin{bmatrix} B_1 \\ B_4 \end{bmatrix} + D \\ &= G_p(s). \end{aligned}$$

Distinction of Cases [Benner, Chu '05]

Case 3: $n_2 = n_3 \neq 0$

- $M_k = 0$ for $k \geq 2$.
- By only using orthogonal transformations we obtain $M_1 = \mathcal{N}^{-1}\mathcal{M}$.
- We can again obtain G_p as

$$G_p(s) = \begin{bmatrix} \mathcal{C}_1 & \mathcal{C}_2 \end{bmatrix} \left(s \begin{bmatrix} \mathcal{E}_{11} & \mathcal{E}_{12} \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} \\ 0 & \mathcal{A}_{22} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathcal{B}_1 \\ \mathcal{B}_2 \end{bmatrix} + \mathcal{D}.$$

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Discussion

- Pro: only using orthogonal transformations, $\mathcal{O}(n^3)$ flops,
- Con: URV and RRQR decompositions with delicate rank decisions.

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Bounded/Positive Realness and Hamiltonian Pencils

Let G_p be a proper and stable with realization $(E; A, B, C, D)$.

Theorem

Let $\lim_{\omega\to\infty} \sigma_{\max}(G(\mathrm{i}\omega)) < 1$. Then G_p is strictly bounded real if and only if the extended skew-Hamiltonian/Hamiltonian (sH/H) matrix pencil

$$\lambda \left[\begin{array}{cc|cc} E & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & E^T & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] - \left[\begin{array}{cc|cc} A & B & 0 & 0 \\ C & D & 0 & I_m \\ \hline 0 & 0 & -A^T & -C^T \\ 0 & -I_m & -B^T & -D^T \end{array}\right],$$

has no finite, purely imaginary eigenvalues.

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Let G_p be a proper and stable with realization $(E; A, B, C, D)$.

Theorem

Let $D + D^T$ be nonsingular and $\lim_{\omega \to \infty} \lambda_{\min}(H(\mathrm{i}\omega)) > 0$. Then G_p is strictly positive real if and only if the extended skew-Hamiltonian/Hamiltonian (sH/H) matrix pencil

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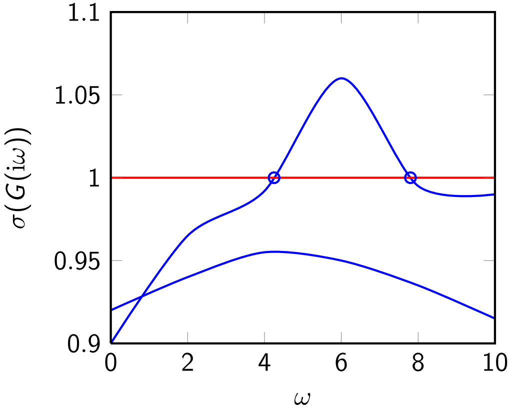
General Idea: If the matrix pencils have purely imaginary eigenvalues, perturb these away from the imaginary axis to passify the proper subsystem.

Graphical Interpretation



Passivation Steps:

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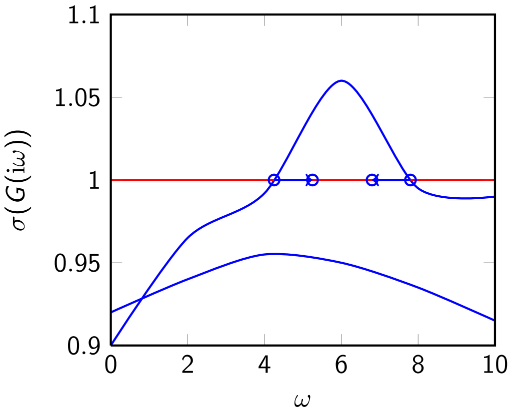

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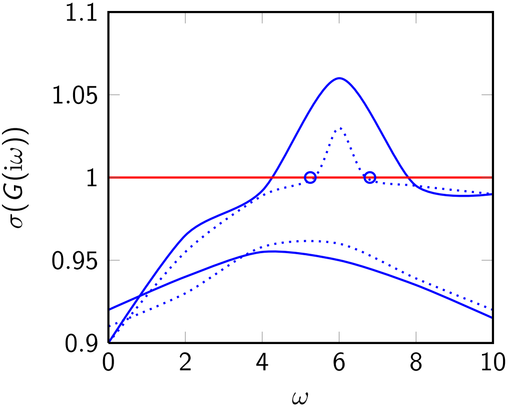

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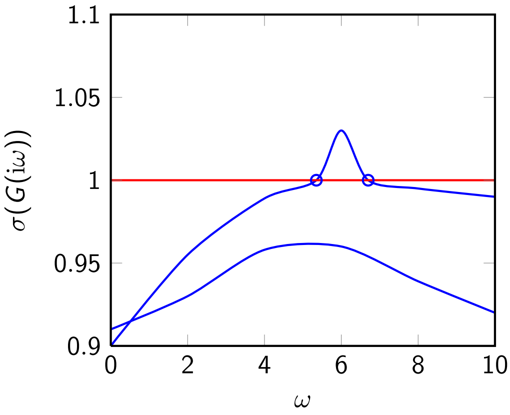

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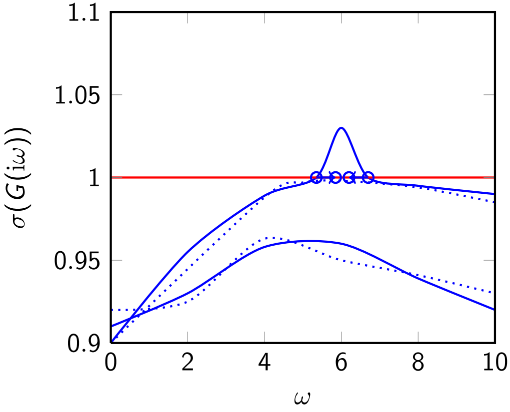

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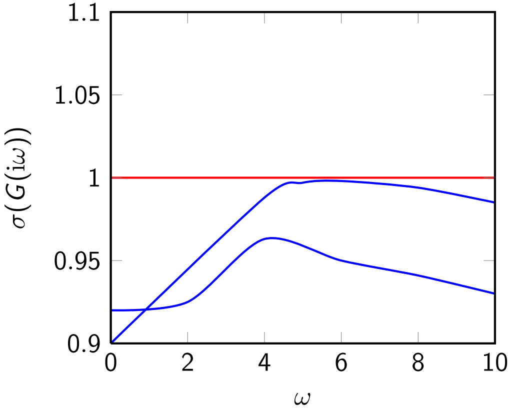

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Choice of the Perturbation

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&= \begin{bmatrix} C_1 & C_2 \end{bmatrix} \left(s \begin{bmatrix} E_{11} & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix} \right)^{-1} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} + D \\
&= \underbrace{C_1 \left(sE_{11} - A_{11} \right)^{-1} B_1}_{=G_{sp}(s)} + \underbrace{D - C_2 A_{22}^{-1} B_2}_{=M_0}
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Which matrix should be perturbed?

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Which matrix should be perturbed?

- Keep $\lambda E_{11} - A_{11}$ to preserve poles of the system $\Longrightarrow$ stability preservation,
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- Keep $\lambda E_{11} - A_{11}$ to preserve poles of the system $\Longrightarrow$ stability preservation,
- keep $A_{22},\ B_2,\ C_2,\ D$ to preserve behaviour for large frequencies,
- free parameters $B_1,\ C_1$.

Choice of the System Norm

We choose the $\mathcal{H}_2$-norm to measure the error, i.e.,

$$\|\mathcal{E}_{sp}\|_{\mathcal{H}_2} := \left\|\tilde{G}_{sp} - G_{sp}\right\|_{\mathcal{H}_2} = \left(\frac{1}{2\pi}\int_{-\infty}^{\infty} \|\mathcal{E}_{sp}(\mathrm{i}\omega)\|_F^2 \,\mathrm{d}\omega\right)^{1/2}.$$

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Theorem [Stykel '06]

Let $\mathcal{G}_o$ be the unique, positive definite solution of the generalized Lyapunov equation

$$E_{11}^T \mathcal{G}_o A_{11} + A_{11}^T \mathcal{G}_o E_{11} = -C_1^T C_1,$$

and $\mathcal{G}_o = R^T R$ a Cholesky factorization. Then

$$\|\mathcal{E}_{sp}\|_{\mathcal{H}_2} = \|R\Delta\|_F \quad \text{with} \quad \Delta := \tilde{B}_1 - B_1.$$

Choice of the System Norm

Remark

In the case that the transfer function is given as

$$G_p(s) = \begin{bmatrix} \mathcal{C}_1 & \mathcal{C}_2 \end{bmatrix} \left(s \begin{bmatrix} \mathcal{E}_{11} & \mathcal{E}_{12} \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} \\ 0 & \mathcal{A}_{22} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathcal{B}_1 \\ \mathcal{B}_2 \end{bmatrix} + \mathcal{D},$$

it is not necessary to decouple G_p into G_{sp} and M_0, since all required matrices are already available!

Computation of the Optimal Perturbation

Let $\lambda\mathcal{N} - \mathcal{M}$ be the sH/H matrix pencil associated to a system in scattering representation (similar for admittance/impedance representation)

$$\lambda\mathcal{N} - \mathcal{M} = \lambda \left[\begin{array}{ccc|ccc} \mathcal{E}_{11} & \mathcal{E}_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \mathcal{E}_{11}^T & 0 & 0 \\ 0 & 0 & 0 & \mathcal{E}_{12}^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] -$$

$$\left[\begin{array}{ccc|ccc} \mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{B}_1 & 0 & 0 & 0 \\ 0 & \mathcal{A}_{22} & \mathcal{B}_2 & 0 & 0 & 0 \\ \mathcal{C}_1 & \mathcal{C}_2 & \mathcal{D} & 0 & 0 & I_m \\ \hline 0 & 0 & 0 & -\mathcal{A}_{11}^T & 0 & -\mathcal{C}_1^T \\ 0 & 0 & 0 & -\mathcal{A}_{21}^T & -\mathcal{A}_{22}^T & -\mathcal{C}_2^T \\ 0 & 0 & -I_m & -\mathcal{B}_1^T & -\mathcal{B}_2^T & -\mathcal{D}^T \end{array}\right].$$

Computation of the Optimal Perturbation

The perturbed pencil is $\lambda\mathcal{N} - \left(\mathcal{M} + \hat{\mathcal{M}}\right)$ with

$$\hat{\mathcal{M}} = \left[\begin{array}{ccc|ccc} 0 & 0 & \Delta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\Delta^T & 0 & 0 \end{array}\right].$$

Computation of the Optimal Perturbation

$$\mathcal{J}\hat{\mathcal{M}} = \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\Delta^T & 0 & 0 \\ \hline 0 & 0 & -\Delta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right], \quad \mathcal{J} = \begin{bmatrix} 0 & I_{n+m} \\ -I_{n+m} & 0 \end{bmatrix}$$

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First Order Perturbation of Eigenvalues

Let $\mathrm{i}\omega_j$ be a simple, finite, purely imaginary eigenvalue of $\lambda\mathcal{N} - \mathcal{M}$ with eigenvector v_j. Then

$$\tilde{\omega}_j - \omega_j = \frac{v_j^H \mathcal{J}\hat{\mathcal{M}} v_j}{\mathrm{i} v_j^H \mathcal{J}\mathcal{N} v_j} + \mathcal{O}\left(\|\hat{\mathcal{M}}\|_2^2\right), \quad j = 1, \ldots, k.$$

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Computation of $v_j \Longrightarrow$ see talk at GAMM Annual Meeting in Graz!

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$$= -\frac{v_{j3}^H \Delta^T v_{j4} + v_{j4}^H \Delta v_{j3}}{\mathrm{i} v_j^H \mathcal{J} \mathcal{N} v_j}$$

Computation of the Optimal Perturbation

$$\begin{aligned}\tilde{\omega}_j - \omega_j &= \frac{v_j^H \mathcal{J} \hat{\mathcal{M}} v_j}{\mathrm{i} v_j^H \mathcal{J} \mathcal{N} v_j} \\ &= -\frac{v_{j3}^H \Delta^T v_{j4} + v_{j4}^H \Delta v_{j3}}{\mathrm{i} v_j^H \mathcal{J} \mathcal{N} v_j} \\ &= -\frac{2\,\mathrm{Re}\left(v_{j4}^H \Delta v_{j3}\right)}{\mathrm{i} v_j^H \mathcal{J} \mathcal{N} v_j}\end{aligned}$$

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&= -\frac{2 \operatorname{Re}\left(v_{j3}^T \otimes v_{j4}^H\right) \operatorname{vec}(\Delta)}{\mathrm{i} v_j^H \mathcal{J} \mathcal{N} v_j}
\end{aligned}$$

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Doing this for all imaginary eigenvalues, we obtain

$$Z\,\mathrm{vec}(\Delta) = \tilde{\omega} - \omega.$$

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Minimize $\|\mathcal{E}_{sp}\|_{\mathcal{H}_2}$ by solving the minimization problem

$$\min_{\Delta \in \mathbb{R}^{m \times n_f}} \|R\Delta\|_F \quad \text{subject to} \quad Z\,\mathrm{vec}(\Delta) = \tilde{\omega} - \omega.$$

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Solution: $\mathrm{vec}(\Delta_R) = Z_R^{\dagger}(\tilde{\omega} - \omega), \quad \Delta = R^{-1}\Delta_R.$

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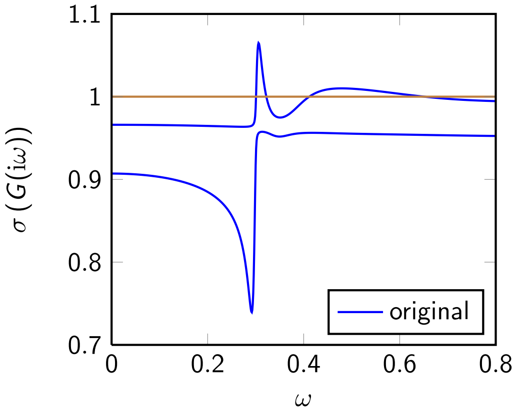
Example Description

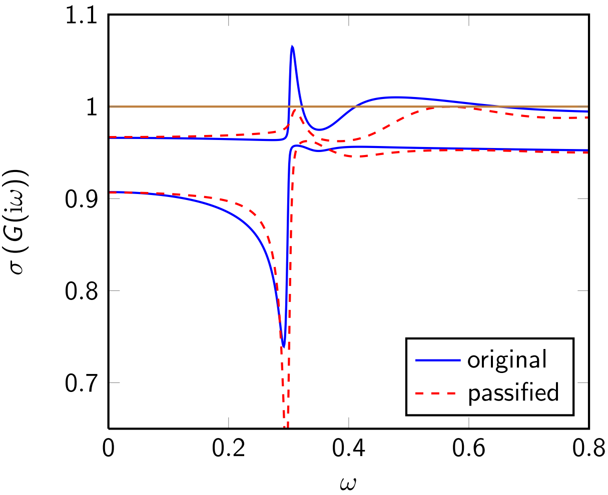
- artificial example system in scattering representation with $n = 12$, $m = 2$,
- passivity violation in $[0.300808, 0.322311] \cup [0.411681, 0.651213]$,
- $\|G\|_{\mathcal{H}_2} = 1.3686$, $\|G\|_{\mathcal{H}_\infty} = 1.0553$.



Some Results

Facts:

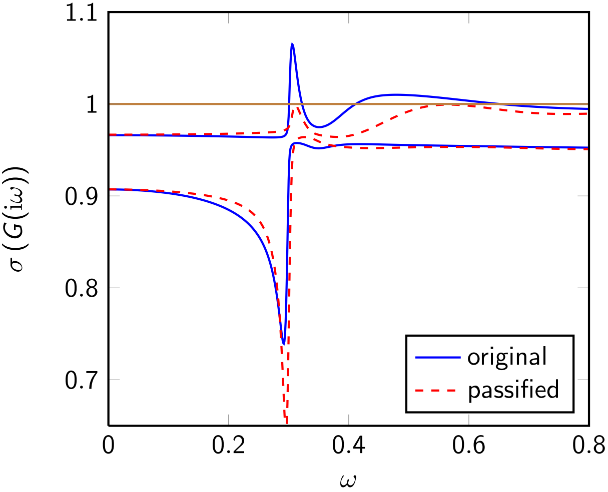
- $\alpha = 0.5$
- $\varepsilon_r = 0.01750$
- iter.: 2



Some Results

Facts:

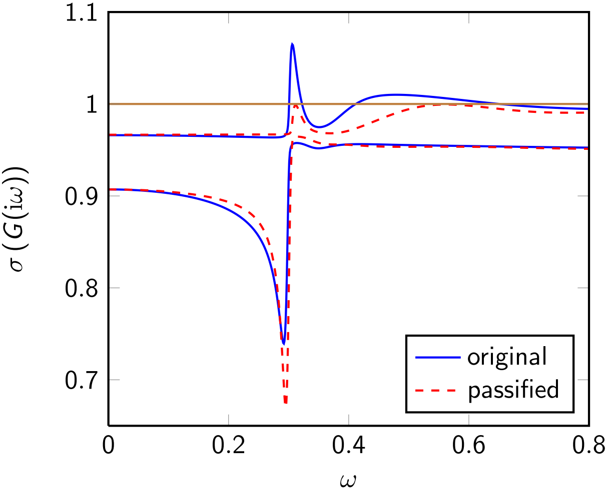
- $\alpha = 0.4$
- $\varepsilon_r = 0.01429$
- iter.: 2



Some Results

Facts:

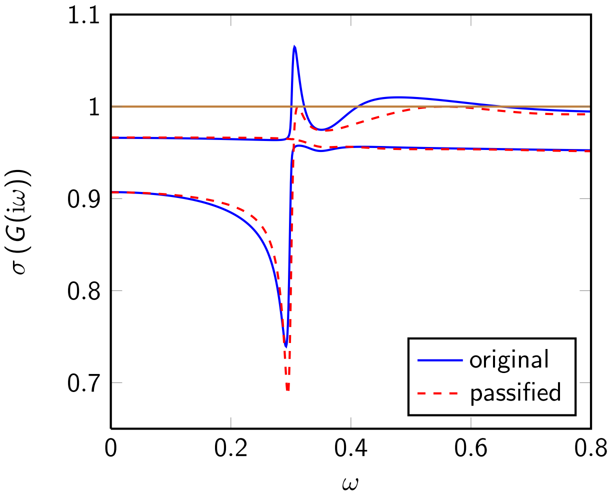
- $\alpha = 0.3$
- $\varepsilon_r = 0.01074$
- iter.: 2



Some Results

Facts:

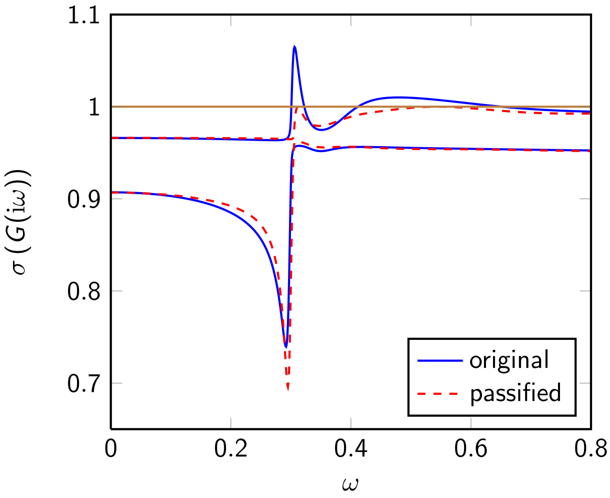
- $\alpha = 0.2$
- $\varepsilon_r = 0.00805$
- iter.: 21



Some Results

Facts:

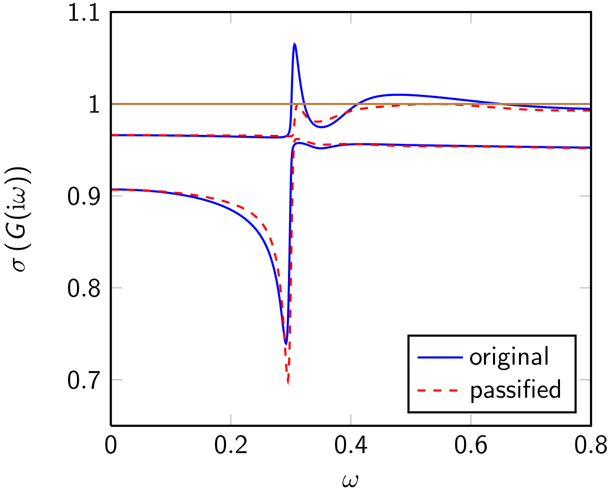
- $\alpha = 0.1$
- $\varepsilon_r = 0.00694$
- iter.: 30



Some Results

Facts:

- $\alpha = 0.05$
- $\varepsilon_r = 0.00665$
- iter.: 72



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Conclusion and Outlook

What we have done

- Characterization of passivity for systems in scattering and admittance/impedance representation in terms of the transfer functions,
- computation of relevant subsystems and Markov parameters,
- passivity enforcement of the proper part using perturbations of sH/H matrix pencils.

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- Asymptotic passivity enforcement for passivity violations at $\omega = \infty$,
- enforcing positive semidefiniteness of M_1. [Wang et al. '10]

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- Asymptotic passivity enforcement for passivity violations at $\omega = \infty$,
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Further research directions

- Large-scale systems,
- consider other matrices than B_1 for perturbation.

Thank you!

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