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Robust and Efficient Algorithms for $\mathcal{L}_\infty$-Norm Computation of Descriptor Systems

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Descriptor Systems

Given: LTI descriptor system

- in continuous-time:

$$\Sigma_c : \quad \begin{cases} E\dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t) + Du(t), \end{cases}$$

- in discrete-time:

$$\Sigma_d : \quad \begin{cases} Ex(t+1) = Ax(t) + Bu(t), \\ y(t) = Cx(t) + Du(t), \end{cases}$$

- $E, A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $D \in \mathbb{R}^{p\times m}$,
- descriptor vector $x(t) \in \mathbb{R}^n$, input vector $u(t) \in \mathbb{R}^m$, output vector $y(t) \in \mathbb{R}^p$.
- Assumptions: $\lambda E - A$ is regular, i.e. $\det(\lambda E - A) \not\equiv 0$.

Frequency Domain Analysis

Transfer Functions

$$G(\lambda) := C\left(\lambda E - A\right)^{-1} B + D,$$

where λ is the Laplace variable s/Z transform variable z.

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Definition: the spaces $\mathcal{L}_\infty^{p\times m}$ and $\mathcal{RL}_\infty^{p\times m}$

- By $\mathcal{L}_{\infty,c}^{p\times m}$ ($\mathcal{L}_{\infty,d}^{p\times m}$) we denote the space of $p \times m$ matrix-valued transfer functions which are bounded on the imaginary axis (the unit circle).
- By $\mathcal{RL}_{\infty,c}^{p\times m}$ ($\mathcal{RL}_{\infty,d}^{p\times m}$) we denote the corresponding rational subspaces. All its elements have a descriptor system realization of the form Σ_c (Σ_d).

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Definition: $\mathcal{L}_\infty$-norm

- in continuous time: $\|G\|_{\mathcal{L}_\infty} := \sup_{\omega\in[0,\infty)} \sigma_{\max}\left(G\left(\mathrm{i}\omega\right)\right);$
- in discrete time: $\|G\|_{\mathcal{L}_\infty} := \sup_{\omega\in[-\pi,\pi)} \sigma_{\max}\left(G\left(e^{\mathrm{i}\omega}\right)\right).$

For stable systems these are equivalent to the $\mathcal{H}_\infty$-norm!

Applications

Model order reduction

Let $\hat{\Sigma} = (\lambda\hat{E} - \hat{A}, \hat{B}, \hat{C}, \hat{D})$ be a reduced order model of the system Σ. The transfer function of the error system $\Sigma^{err} := \Sigma - \hat{\Sigma}$ is given by

$$G^{err}(\lambda) = \begin{bmatrix} C & -\hat{C} \end{bmatrix} \left(\lambda \begin{bmatrix} E & 0 \\ 0 & \hat{E} \end{bmatrix} - \begin{bmatrix} A & 0 \\ 0 & \hat{A} \end{bmatrix} \right)^{-1} \begin{bmatrix} B \\ \hat{B} \end{bmatrix} + D - \hat{D}.$$

$\|G^{err}\|_{\mathcal{H}_\infty}$ is the size of the worst-case approximation error.

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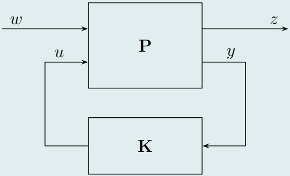
$\mathcal{H}_\infty$-control [Green, Limebeer '95]



- Plant **P**, dynamic compensator **K**,
- noise w, estimation error z,
- $\mathcal{H}_\infty$-norm of the transfer function from w to z displays the worst-case influence of the disturbances w on the output z.

Basic Theorems

Theorem [Genin, Van Dooren, Vermaut '98, V '10]

1. Assume that $\lambda E - A$ has no purely imaginary eigenvalues, $G \in \mathcal{RL}_{\infty,c}^{p \times m}$, $\gamma > 0$ is not a singular value of D, and $\omega_0 \in \mathbb{R}$. Then, γ is a singular value of $G(\mathrm{i}\omega_0)$ if and only if

$$H_c(\gamma) := \begin{bmatrix} \lambda E - A & 0 \\ 0 & \lambda E^T + A^T \end{bmatrix} - \begin{bmatrix} B & 0 \\ 0 & -C^T \end{bmatrix} \begin{bmatrix} -D & \gamma I_p \\ \gamma I_m & -D^T \end{bmatrix}^{-1} \begin{bmatrix} C & 0 \\ 0 & B^T \end{bmatrix}$$

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has the eigenvalue $\mathrm{i}\omega_0$.

2. Assume that $\lambda E - A$ has no unitary eigenvalues, $G \in \mathcal{RL}_{\infty,d}^{p\times m}$, $\gamma > 0$ is not a singular value of D, and $\omega_0 \in [-\pi, \pi)$. Then, γ is a singular value of $G\left(e^{\mathrm{i}\omega_0}\right)$ if and only if

$$H_d(\gamma) := \begin{bmatrix} \lambda E - A & 0 \\ 0 & \lambda A^T - E^T \end{bmatrix} - \begin{bmatrix} B & 0 \\ 0 & -\lambda C^T \end{bmatrix} \begin{bmatrix} -D & \gamma I_p \\ \gamma I_m & -D^T \end{bmatrix}^{-1} \begin{bmatrix} C & 0 \\ 0 & B^T \end{bmatrix}$$

has the eigenvalue $e^{\mathrm{i}\omega_0}$.

Outline of the Algorithm

Corollaries

1. Assume that $G \in \mathcal{RL}_{\infty,c}^{p\times m}$ and let $\gamma > \min_{\omega\in\mathbb{R}} \sigma_{\max}(G(\mathrm{i}\omega))$ be not a singular value of D. Then $\|G\|_{\mathcal{L}_\infty} \geq \gamma$ if and only if $H_c(\gamma)$ has finite, purely imaginary eigenvalues.
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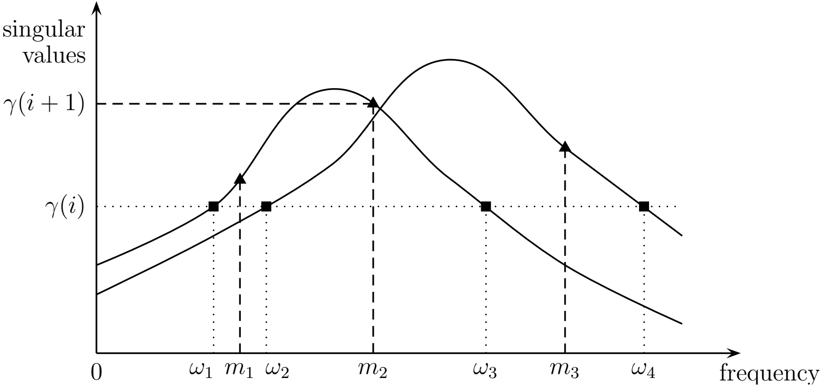
Sketch of the Algorithm

1. Choose initial value of γ.
2. Check $H_c(\gamma)$ ($H_d(\gamma)$) for imaginary (unitary) eigenvalues.
3. If imaginary (unitary) eigenvalues, then increase γ, else $\|G\|_{\mathcal{L}_\infty}$ is found.

Graphical Interpretation



The Eigenvalue Problems

Reminder: original matrix pencil (continuous-time case)

$$H_c(\gamma) := \begin{bmatrix} \lambda E - A & 0 \\ 0 & \lambda E^T + A^T \end{bmatrix} - \begin{bmatrix} B & 0 \\ 0 & -C^T \end{bmatrix} \begin{bmatrix} -D & \gamma I_p \\ \gamma I_m & -D^T \end{bmatrix}^{-1} \begin{bmatrix} C & 0 \\ 0 & B^T \end{bmatrix}$$
$$= \begin{bmatrix} \lambda E - A + BR_\gamma^{-1} D^T C & \gamma B R_\gamma^{-1} B^T \\ -\gamma C^T S_\gamma^{-1} C & \lambda E^T + A^T - C^T D R_\gamma^{-1} B^T \end{bmatrix}$$

with $R_\gamma := DD^T - \gamma^2 I_p$ and $S_\gamma := D^T D - \gamma^2 I_m$.

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- Forming "matrix-times-its-transpose" products numerically unstable
  $\Longrightarrow$ explicitly forming $H_c(\gamma)$ must be avoided!

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Remarks

- If γ is close to a singular value of D, then R_γ, S_γ are ill-conditioned.
- Forming "matrix-times-its-transpose" products numerically unstable $\Longrightarrow$ explicitly forming $H_c(\gamma)$ must be avoided!
- $H_c(\gamma)$ is a skew-Hamiltonian/Hamiltonian pencil $\Longrightarrow$ structure-preserving algorithms?

Matrix Structures

Even pencils

A real $n \times n$ matrix pencil $\lambda S - H$ is called even, if S is skew-symmetric and H is symmetric.

Skew-Hamiltonian/Hamiltonian pencils

Let $\mathcal{J} = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix}$. A real $2n \times 2n$ matrix pencil $\lambda \mathcal{S} - \mathcal{H}$ is called skew-Hamiltonian/Hamiltonian if $\mathcal{S}$ is skew-Hamiltonian ($(\mathcal{S}\mathcal{J})^T = -\mathcal{S}\mathcal{J}$) and $\mathcal{H}$ is Hamiltonian ($(\mathcal{H}\mathcal{J})^T = -\mathcal{H}\mathcal{J}$).

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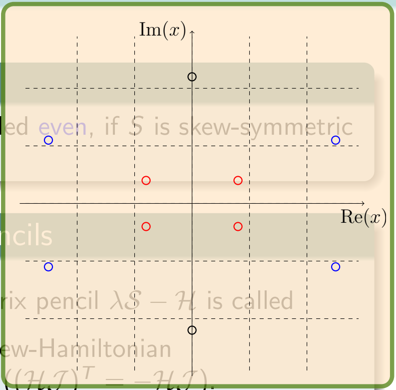

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Properties

- Hamiltonian eigensymmetry,
- easy to switch between the two types.

Transformation of $H_c(\gamma)$

Reminder: original matrix pencil

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Exploit Schur complement structure and get the extended matrix pencil

$$\mathcal{H}_c^{(1)}(\gamma) = \left[\begin{array}{cc|cc} \lambda E - A & 0 & -B & 0 \\ 0 & \lambda E^T + A^T & 0 & C^T \\ \hline -C & 0 & -D & \gamma I_p \\ 0 & -B^T & \gamma I_m & -D^T \end{array}\right],$$

which has the same eigenvalues as $H_c(\gamma)$ with additional infinite eigenvalues.

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which has the same eigenvalues as $H_c(\gamma)$ with additional infinite eigenvalues. Block permutations yield the even pencil

$$\mathcal{H}_c^{(2)}(\gamma) = \left[\begin{array}{cc|cc} 0 & \lambda E - A & 0 & -B \\ -\lambda E^T - A^T & 0 & -C^T & 0 \\ \hline 0 & -C & \gamma I_p & -D \\ -B^T & 0 & -D^T & \gamma I_m \end{array}\right].$$

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which has the same eigenvalues as $H_c(\gamma)$ with additional infinite eigenvalues. Block permutations yield the D-type pencil

$$\mathcal{H}_d^{(2)}(\gamma) = \left[\begin{array}{c|ccc} 0 & -\lambda E^T + A^T & C^T & 0 \\ \hline \lambda A - E & 0 & 0 & -B \\ \lambda C & 0 & \gamma I_p & -D \\ 0 & -B^T & -D^T & \gamma I_m \end{array}\right].$$

Transformation to an Even Pencil

Given: D-type pencil

$$\lambda \mathcal{E}_D - \mathcal{A}_D = \lambda \begin{bmatrix} 0 & F \\ -G^T & 0 \end{bmatrix} - \begin{bmatrix} 0 & G \\ -F^T & H \end{bmatrix}, \quad H = H^T$$

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Application of Cayley + drop/add transformations [Xu '06]

$$\lambda\mathcal{E}_C - \mathcal{A}_C = \lambda \begin{bmatrix} 0 & G+F \\ -G^T - F^T & 0 \end{bmatrix} - \begin{bmatrix} 0 & G-F \\ G^T - F^T & H \end{bmatrix}$$

- $\lambda\mathcal{E}_C - \mathcal{A}_C$ is a C-type pencil (which is an even pencil),
- Hamiltonian eigensymmetry,
- unitary eigenvalues of $\lambda\mathcal{E}_D - \mathcal{A}_D$ mapped to imaginary eigenvalues of $\lambda\mathcal{E}_C - \mathcal{A}_C$.

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We obtain

$$\mathcal{H}_d^{(3)}(\gamma) = \lambda \left[\begin{array}{c|ccc} 0 & -A^T - E^T & -C^T & 0 \\ \hline A+E & 0 & 0 & 0 \\ C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] - \left[\begin{array}{c|ccc} 0 & -A^T + E^T & -C^T & 0 \\ \hline -A+E & 0 & 0 & B \\ -C & 0 & -\gamma I_p & D \\ 0 & B^T & D^T & -\gamma I_m \end{array}\right].$$

Remarks on Software

Structure-Preserving Algorithms

- use special eigensolver for skew-Hamiltonian/Hamiltonian pencils that respects the structure, [Benner, Byers, Mehrmann, Xu '99]
- robust detection of purely imaginary eigenvalues (no error in real parts).

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Implementation

- efficient Fortran 77 implementation for the Control Library SLICOT,
- often shows better performance and robustness compared to standard software like MATLAB,
- upcoming routine `AB13HD` for norm computation with structured eigensolver `MB04BD`.

A Discrete-Time Toy Example

Example description

- $n = 11$ descriptor variables, $m = 1$ input, $p = 1$ output,
- $\lambda E - A$ has 8 finite and 3 infinite eigenvalues.

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iteration	γ	imag. eig. of $\mathcal{H}^{(3)}(\gamma)$
1	$5.50049659572561002 \cdot 10^{-3}$	1.431624i 674255.553476i
2	$5.57395402021302819 \cdot 10^{-3}$	1.791283i 3.177911i
3	$5.59298056341866415 \cdot 10^{-3}$	2.202331i 2.319735i
4	$5.59316018834055389 \cdot 10^{-3}$	2.258739i 2.259758i
5	$5.59316020191487611 \cdot 10^{-3}$	none

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5	$5.59316020191487611 \cdot 10^{-3}$	none

- $\|G\|_{\mathcal{L}_\infty} = 5.59316020191239199 \cdot 10^{-3}$,
- peak frequency: $\omega_p = 2.3081845795501987$.

Performance

Comparison of modified COMPl$_e$ib examples [Leibfritz, Lipinski '10]

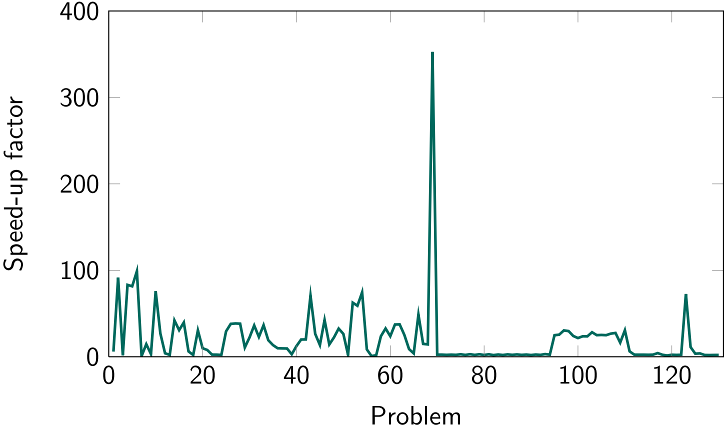

Figure: SLICOT structured solver versus MATLAB `norm` for modified COMPl$_e$ib examples:

Conclusions and Outlook

What we have done

- $\mathcal{L}_\infty$-norm computation for discrete-time descriptor systems by using structured matrix pencils,
- matrix pencil transformations in order to use a structure-preserving algorithm to compute the eigenvalues.

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- $\mathcal{H}_\infty$-norm computation for large-scale systems using structured pseudospectra [Benner, V. '12]

Thank you for your Attention!

References

References

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