Comments and corrections to my book

Optimal control of partial differential equations

February 2016

I thank all readers, who commented on errors or typos in my book. In particular, I am grateful to

Roland Herzog (Chemnitz)

Daniel Arroyo (Berlin)

Luc Paquet (Valenciennes)

Patrick Sodre (Maryland)

for their hints that are considered below.

page/line	Correction
17_6	$i \in \{a, b\}$
25^8	In part (ii) of the definition, the function v must be continuous in Ω.
56^{13}	In this definition, the Limes $\lim_{t \downarrow 0}$ refers only to the directional derivative. To define the first variation, the Limes $\lim_{t \to 0}$ has to be used.
56_{11}	The correct definition of f is $f(x) = 3\, r \cos(\varphi)$.
71^1	$a \le b$, $\mathbb{P}_{[a,b]}$ ...
77^6	$y = y - \bar{y}$ and $u = u - \bar{u}$ ...
80^8	$\int_\Omega (\beta_\Omega\, p + \lambda_\Omega\, \bar{v})(v - \bar{v})\, dx + \int_\Gamma (\beta_\Gamma\, p + \lambda_\Gamma\, \bar{u})(u - \bar{u})\, ds \ge 0$
92_4	"In the convex case": This means that f and U_{ad} are convex.
96	In subsection 2.12.3, it is tacitly assumed that all appearing functions are continuous so that terms like $\Delta y(x_{ij})$, $y_\Omega(x_i)$, $u_a(x_i)$ etc. are defined.
104_3	Since D_h is a diagonal matrix with positive diagonal elements ...
124	In Assumption 3.1, the non-negativity $\beta(t) \ge 0$ is not needed.
130^1	$\tilde{p}_\tau(x, \tau) = \tilde{p}_{xx}(x, \tau)$
150_9	The functionals F_i were defined at page 149 without a minus sign. Therefore, we must write in line 150_9: "... being just the sum of the $-F_i$ ..."
152^7	$v \in W_2^{1,1}(Q)$
209_8	$\{v \in L^2(\Omega) : \|v\|_{L^\infty(\Omega)} \le M\}$
227^5	It should be mentioned that $F''(u)[u_1, u_2]$ is a bilinear form with values in V.
233^2	We have $2 > 3/2 \ge N/2$ ...
233^7	The second-order Fréchet differentiability of G is true but not yet clear at this page. In Theorem 4.24, this property is shown for $G : L^\infty(\Omega) \to H^1(\Omega) \cap C(\bar{\Omega})$ by means of the implicit function theorem. Roughly speaking the implicit function theorem ensures that the differentiability of the implicit function is the same as the one of the given nonlinearities. We already know that G is continuously Fréchet differentiable from $L^r(\Omega)$ to $H^1(\Omega) \cap C(\bar{\Omega})$ provided that $r > N/2$. Therefore, also the second-order differentiability should hold in this pair of spaces. This is true; we refer to Section 4.10.6, "Cases without two-norm discrepancy", where these facts are briefly discussed at page 255.
245	In the third line of the subsection "Second-order necessary conditions", the term "yields for the solution to problem (4.31)–(4.33)" can be replaced by the more general formulation "yields for any local solution to problem (4.31)–(4.33)"
246	In Theorem 4.27, a locally optimal control in the sense of $L^\infty(\Omega)$ is considered.
256	In the lines 256_9 and 256_1, the variable λ must be deleted. The correct form of ψ is $\psi(x, u) = \gamma_1(x)\, u + \gamma_2(x)\, u^2$.
332_{10}	The brackets in this formula are not correct. The correct version is $\alpha\, G'(\bar{u})(u - \bar{u}) + \beta\, (k + G(\bar{u})) = z.$
338^5	$(\bar{y}, \bar{v})$ satisfies ...
343	In the remark to the application of the formal Lagrangian technique, the knowledge is tacitly assumed that the Lagrange multiplier μ associated with the state constraint is a measure.