Technische Universität Berlin – Institut für Mathematik
Prof. Dr. Reinhold Schneider

WiSe 2014/15

NUMERICS OF PARTIAL DIFFERENTIAL EQUATIONS

Series 5

1. Solve the equation $\Delta u = f$ for some inhomogeneous Dirichlet conditions of your choice. In order to do this write a file u_d.m with the function y=u_d(x) where x is an N-by-2 matrix containing the coordinates of the boundary points and y is an N-by-1 matrix with the Dirichlet boundary values. Next, modify your finite element code to include these boundary values using the method discussed in the lecture of writing the solution of the inhomogeneous system as a solution of a homogeneous modified system and an extension of the boundary conditions to Ω. Can you give an explicit upper bound on the error of your finite element solution?

2. Next, generate a mesh using the tool described in triangle_shortref.pdf. In particular you can generate a mesh representing a frying pan. Create an L-shaped mesh and compare the convergence to the one you get when the domain is a square or circular mesh. Create a slit domain and investigate convergence experimentally.

3. Consider a rotationally symmetric inhomogeneous Dirichlet boundary value problem in three dimensions $\Delta u = 0$, $u|_\Omega = g$. Can you give an equivalent problem, that only needs two dimensions.

4. Calculate some problem with Neumann boundary conditions. For example the two dimensional problem from exercise 3. For this it might be convenient to change the generation of dirichlet.dat to output a vector containing only the edges on which you want to impose dirichlet conditions. It makes sense that you split the boundary in one part where you impose Dirichlet conditions and another part where you impose Neumann conditions. Try what happens if you set Neumann conditions on the whole boundary and nowhere Dirichlet conditions. Would the problem have a unique solution in this case? If not: Which assumption of the inf-sup-conditions would we fail to fulfill? Also try what happens if you set Dirichlet conditions on every second edge and Neumann conditions on every other.

See next page!

5. Solve the problem from exercise 3 in three dimensions by modifying your finite element code. Look at the C/A/F paper for details.

To be handed in by: 2015 (2.00 pm)
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