Technische Universität Berlin – Institut für Mathematik WiSe 2014/15
Prof. Dr. Reinhold Schneider

NUMERICS OF PARTIAL DIFFERENTIAL EQUATIONS

Series 6

1. (easy) Consider the homogeneous Dirichlet boundary value problem in two dimensions $\Delta u = 0$, $u|_\Omega = 0$. Get the CG-code from your exercise notes or wikipedia and replace the matlab-routine for solving the matrix equation. Output the solution after each iteration of the CG-algorithm. Compare your results to the GIF's on the course website. By looking at these pictures, do you have an idea, why the convergence is so slow and how to improve it?

2. (bit tricky to implement) This exercise is based on the 1986 paper 'On the Multi-Level Splitting of Finite Element Spaces' by Harry Yserentant. Solve the problem above using hierarchical finite elements. Hierarchical finite elements means replacing some basis functions from a fine grid by basis functions of a coarser grid. Doing this in the right way preserves the discretization space of the fine grid. As you can express the coarser basis function as a weighted sum of the fine basis functions, you can build a matrix S to do this (tricky part). Doing the procedure recursively until you arrive at the coarsest basis functions generates you a series of such matrices. You can now write the stiffness matrix for the hierarchical basis as $S^T AS$. Again, plot all the steps of the CG-algorithm and compare the convergence and pictures with the results from exercise 1. Also calculate the eigenvalues and condition numbers of the hiearchical stiffness matrix using the matlab function $eig(...)$. The eigenvalues might depend on the scaling of your hierarchical basis functions.

3. Show that the CG-method is a Krylov-subspace method.

4. Derive a relation for the n+1 height vectors in the n-simplex. Hint: Use shape functions and gradients.

To be handed in by: 2015 (2.00 pm)
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