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WS 2016/17

1. Exercise “Differential-Algebraic Equations“

(Due date: 31.10.2016)

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1. Let $\mathbb{I} \subset \mathbb{R}$ be an interval and $f : \mathbb{I} \times \mathbb{R}^n \to \mathbb{R}^n$ a globally Lipschitz continuous function. Show that: For all $t \in \mathbb{I}$ there exists a $c_t > 0$, such that for all $\delta(\cdot) \in C(\mathbb{I}, \mathbb{R}^n)$, $x_0, x_{0\delta} \in \mathbb{R}^n$ holds: the solutions of $\dot{x}(t) = f(t, x(t))$, $x(t_0) = x_0$, $\dot{x}_\delta(t) = f(t, x_\delta(t)) + \delta(t)$, $x_\delta(t_0) = x_{0\delta}$ satisfy the inequality

$$\|x(t) - x_\delta(t)\| \le c_t \cdot \left(\|x_0 - x_{0\delta}\| + \int_{t_0}^{t} \|\delta(\tau)\| d\tau \right).$$

2. Consider the differential-algebraic equation

$$\begin{aligned} \dot{x}_1(t) - x_1(t) &= 0 \\ x_1(t)^2 - x_2(t) + f(t) &= 0 \\ x_3(t) - \dot{x}_2(t) + g(t) &= 0. \end{aligned}$$

(a) Give a general solution of the system for $x_1(t), x_2(t), x_3(t)$.

(b) Which smoothness requirements have to be satisfied by $f(\cdot), g(\cdot)$ such that a solution exists.

(c) Determine the set of consistent initial values $x_1(0), x_2(0), x_3(0)$.

(d) Show that an estimate as in 1. is not possible.

3. Check whether the matrix pairs

$$\left(\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} \right), \quad \left(\begin{bmatrix} 2 & -1 & 1 \\ 3 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \right)$$

are regular or singular. For regular matrix pairs determine the Weierstraß canonical form and the index $\operatorname{ind}(E, A)$.