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3. Exercise "Differential-Algebraic Equations"

(Due date: 28.11.2016)

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1. Exercise

For which inhomogeneities $f \in C(\mathbb{R}, \mathbb{C}^m)$ are the following DAEs solvable with $x \in C^1(\mathbb{R}, \mathbb{C}^n)$?

(a) $m = n = 1$, $t\dot{x} = f(t)$,

(b) $m = n = 1$, $0 = tx + f(t)$,

(c) $m = n = 2$, $\dot{x}_1 = f_1(t), 0 = tx_1 + f_2(t)$.

2. Exercise

Consider the time-dependent matrix pair

$$(E(t), A(t)) = \left(\begin{bmatrix} 0 & 0 \\ 1 & \eta t \end{bmatrix}, \begin{bmatrix} -1 & -\eta t \\ 0 & -(1+\eta) \end{bmatrix} \right), \quad \eta \in \mathbb{R}.$$

1. Determine the local characteristic quantities (r, a, s) for every $t \in \mathbb{R}$ and every $\eta \in \mathbb{R}$.
2. Determine the global characteristic values (r_i, a_i, s_i), $i = 0, \ldots \mu$ for every $\eta \in \mathbb{R}$.
3. Determine a reduced pair $(\hat{E}(t), \hat{A}(t))$ that is strangeness-free.

3. Exercise

Let $A \in \mathbb{C}^{m,k}$ and $B \in \mathbb{C}^{k,n}$ have full row rank. Prove that then AB has full row rank.