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4. Exercise "Differential-Algebraic Equations"

(Due date: 05.01.2017)

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1. Exercise

Consider the following DAEs:

$$\begin{aligned} \dot{q} &= v \\ M\dot{v} &= Fq - G^T\lambda \\ 0 &= Gq - r \end{aligned} \qquad (1) \qquad\qquad \begin{aligned} \dot{q} &= v - G^T\eta \\ M\dot{v} &= Fq - G^T\lambda \\ 0 &= Gv \\ 0 &= Gq - r \end{aligned} \qquad (2)$$

with $q : \mathbb{I} \to \mathbb{R}^{n_q}$, $v : \mathbb{I} \to \mathbb{R}^{n_v}$, $\lambda : \mathbb{I} \to \mathbb{R}^{n_\lambda}$, and $\eta : \mathbb{I} \to \mathbb{R}^{n_\eta}$. M, F and G are matrices of suitable size, with M positive definite and G of full row rank. Let r be a vector of suitable size.

1. Show that: $(q^*, v^*, \lambda^*, \eta^*)$ with $\eta^* = 0$ is a solution of (2) if and only if (q^*, v^*, λ^*) is a solution of (1).
2. Determine the strangeness index μ of (1) and (2).

2. Exercise

Determine the differentiation index ν_d and an underlying ODE for the following DAEs

(a)

$$\begin{aligned} \dot{x}_1 &= -x_3 + x_4 \\ \dot{x}_3 &= -x_1 + x_2 \\ 0 &= x_1 - p(t) \\ 0 &= x_4 - q(t) \end{aligned}$$

(b)

$$\begin{aligned} \dot{x}_2 &= -x_3 + x_4 \\ \dot{x}_4 &= -x_1 + x_2 \\ 0 &= x_1 - p(t) \\ 0 &= x_4 - q(t) \end{aligned}$$

(c)

$$\begin{aligned} \dot{x}_1 &= -x_3 + x_4 \\ \dot{x}_4 &= -x_1 + x_2 \\ 0 &= x_1 - p(t) \\ 0 &= x_4 - q(t) \end{aligned}$$

(d)

$$\begin{aligned} \dot{x}_2 &= -x_3 + x_4 \\ \dot{x}_3 &= -x_1 + x_2 \\ 0 &= x_1 - p(t) \\ 0 &= x_4 - q(t) \end{aligned}$$

3. Exercise

Consider the linear DAE

$$E\dot{x} = Ax + f(t), \quad E, A \in \mathbb{R}^{n,n},\ f \in C(\mathbb{I}, \mathbb{R}^n).$$

Let E be symmetric and A positive definite. Show that then the DAE has differentiation index ν_d with $\nu_d \leq 1$.