# Project Ideas for the software seminar on Numerical Algebraic Geometry

## 1. Bottlenecks

The size of the smallest bottleneck of an embedded manifold is a measure of how close it is to being self-intersecting. Knowing this is relevant for data analysis, because bottlenecks affect the quality of numerical algorithms for estimating the topology. David Eklund constructed a homotopy with the optimal number of paths (under genericty assumptions) to compute all bottlenecks of an algebraic variety.

Describe the equations and the algorithm and implement it in Julia using HomotopyContinuation.jl.

***Literature:***

* D. Eklund: The numerical algebraic geometry of bottlenecks. arxiv.org/pdf/1804.01015.pdf

## 2. alphaCertified

Numerically solving systems of polynomials means computing numerical approximations of the true zeros. If the approximation is good enough, the sequence of Newton iterates starting at this approximation converges to the true solution. alphaCertified provides a way for certifying the "good enough".

Describe the ideas behind alphaCertified and implement the algorithm in Julia.

***Literature:***

* F. Sottile and J. Hauenstein: alphaCertified: Certifying Solutions to Polynomial Systems. J. ACM Transactions on Mathematical Software (TOMS) 38 (4) 2012.
* www.math.tamu.edu/~sottile/research/stories/alphaCertified/

## 3. The degree of an algebraic variety

The degree of an algebraic variety is an important invariant. It is defined as the number of complex intersection points of the variety with a random linear space of complementary dimension. A way for computing the degree is to compute one point of intersection with a linear space and then using monodromy by rotating the linear space. The trace test is a tool for certifying if one has found all solutions, or if some are still missing.

Describe the ideas behind the monodromy method and implement an algorithm for computing the degree of a variety in Julia using HomotopyContinuation.jl. Develop a suitable data structure for the output of your algorithm.

***Literature:***

* A. Sommese and C. Wampler. The numerical solution of systems of polynomials. Applied Mathematics Series. Scientific Publishing Co. 2005.
* T. Duff, C. Hill, A. Jensen, K. Lee, A. Leykin, J. Sommars: Solving polynomial systems via homotopy continuation and monodromy. arxiv.org/abs/1609.08722
* www.juliahomotopycontinuation.org/guides/monodromy/

## 4. Four-bar linkage

A four-bar linkage is a planar mechanism, which consists of four bars, which are assembled in a chain, such that the first bar and the fourth bar are rigidly joined. The link between the second and the third bar is called a "hand". A central question in kinematics is to determine which points can be reached by the hand. This problem is called the forward-problem. The corresponding backward-problem is to compute the mechanism which can reach a fixed set of points. Alt showed that, if nine points are specified, there is only a finite number of mechanisms that can touch all of them.

Describe Alt's problem and compute a solution using HomotopyContinuation.jl using the monodromy method. If you can, exploit group actions. Visualize your solution. Optional: create an interactive tool, in which the user can specify the nine points by using clicking with the mouse in the plane (for instance, using Makie.jl).

***Literature:***

* en.wikipedia.org/wiki/Four-bar_linkage
* A. Morgan, A. Sommese and C. Wampler. Complete solution of the nine-point path synthesis problem for four-bar linkages. J. Mechanical Design 114.1 (1992).
* T. Duff, C. Hill, A. Jensen, K. Lee, A. Leykin, J. Sommars: Solving polynomial systems via homotopy continuation and monodromy. arxiv.org/abs/1609.08722
* www.juliahomotopycontinuation.org/guides/monodromy/
* github.com/JuliaPlots/Makie.jl

## 5. The Multiview Variety

Think of an object in a 3-dimensional space. Taking a photo of this object can be modeled by a linear projection onto a 2-dimensional space. Accordingly, taking n photos can be modeled by a linear projection onto the cartesian product of n 2-dimensional spaces. The set of all n-tuples of photos of all points is called the n-multiview variety. The reconstruction problem in computer vision is to take a point on that variety and reconstruct the point from which the n photos are taken. However, when data is noisy, the point is not exactly on the multiview variety but close to it.

Discuss the multiview variety. Set up a system of equations for computing a point on the multiview variety that minimizes the distance to a given point. Solve these equations using the monodromy method in HomotopyContinuation.jl. Reconstruct a 3-dimensional object from a dataset of photos.

***Literature:***

* H. Stewenius, F. Schaffalitzky and D. Nister. How hard is 3-view triangulation really? Tenth IEEE International Conference on Computer Vision (ICCV'05). Volume 1.
* J. Draisma, E. Horobet, G. Ottaviani, B. Sturmfels and R. Thomas. The Euclidean Distance Degree of an Algebraic Variety. J. FOCM. 16 (1), 2016.
* L. Maxim, J. Rodriguez and B. Wang. Euclidean distance degree of the multiview variety. arxiv.org/abs/1812.05648

## 6. Your own topic

If you have an idea for another exciting project, we'd love to hear about it.