Low rank hierarchical tensors approximation

May 15, 2014

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Abstract

Hierarchical Tucker tensor format (HT - Hackbusch tensors) and Tensor Trains (TT- Tyrtyshnikov tensors, I.Oseledets) have been introduced recently for low rank tensor product approximation. Perhaps hierarchical tensor representations have been apparent in tensor network states, like matrix product states in quantum physics since several years. Hierarchical tensor decompositions are based on sub space approximation by extending the Tucker decomposition into a multilevel framework. Therefore they inherit favorable properties of Tucker tensors, e.g they offer a stable and robust approximation, but still enabling low order scaling with respect to the dimensions. For many high dimensional problems, hard to be handled so far, this approach may offer a novel strategy to circumvent the curse of dimensionality.

For numerical computations, we consider the solution of convex optimization problems, constraint by the restriction to tensors of prescribed ranks $\mathbf{r}$, and/or the differential equations obtained by the Dirac-Frenkel principle for dynamical problems. Both can be casted into a non-linear Ritz-Galerkin framework. Based on the known projection onto the tangent space, we derive at non-linear but low dimensional differential equations.

We would like to acknowledge contributions from the recently developing numerical math community as well as from the quantum physics community.