# On boundary-value problems for hyperbolic and degenerate parabolic conservation laws

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The talk is devoted to the problems that can be written under the form

$$u_t + \operatorname{div}\Big(f(u) - \nabla\phi(u)\Big) = 0 \;\text{ in } (0,T)\times\Omega$$

with initial condition and different *boundary* conditions (BC). This setting contains both pure hyperbolic conservation laws ($\phi \equiv 0$) and strongly degenerate convection-diffusion equations encountered in some porous medium, sedimentation and road traffic models. So far, only the Dirichlet BC $u = u^D$ on $(0,T)\times\partial\Omega$ has been studied extensively, starting from the celebrated work [3].

Our main goal is to develop general approaches for treating, in a unified way, different boundary conditions: Dirichlet, zero-flux, obstacle,... We are also interested in efficient numerical approximation of such problems.

Because of purely hyperbolic degeneracy, as in the case of [3] the appropriate formulation or the BC should be seen as a singular limit formulation. One can start with a numerical scheme or a vanishing viscosity approximation, where the BC are taken into account in a straightforward way, and derive the formulation satisfied at the limit. At the limit, it may happen that the BC undergo a projection procedure: namely, the limit satisfies an *effective* BC that can differ from the *formal* BC prescribed for the approximations. We attempt to describe this procedure. This goal is achieved in the pure hyperbolic setting in [2].

In the degenerate parabolic setting, the zero-flux BC $(f(u) - \nabla\phi(u))\cdot\nu = 0$ was treated in [1]. This practically important BC will receive a special attention.

## References

[1] B. Andreianov and M. Karimou Gazibo, *Entropy formulation of degenerate parabolic equation with zero-flux boundary condition*, ZAMP, 2013. Available as hal.archives-ouvertes.fr/hal-00697593 preprint.

[2] B. Andreianov and K. Sbihi, *Well-posedness of general boundary-value problems for scalar conservation laws*, Trans. AMS, accepted. Available as hal.archives-ouvertes.fr/hal-00708973 preprint.

[3] C. Bardos, A.Y. Le Roux and J.-C. Nédélec. *First order quasilinear equations with boundary conditions,* Comm. Partial Diff. Equ. 4(4):1017–1034, 1979.