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The peridynamic equation of motion in non-local elasticity theory

Etienne Emmrich*, **Olaf Weckner**†

*Technische Universität Berlin, Institut für Mathematik
Straße des 17. Juni 136, 10623 Berlin, Germany
emmrich@math.tu-berlin.de

†Massachusetts Institute of Technology, Department of Mechanical Engineering
Cambridge, MA 02139, USA
olaf@weckner.de

ABSTRACT

During the last few years, non-local theories in solid mechanics that account for effects of long-range interactions have become topical again. One of these theories is the so-called peridynamic modelling, introduced by Silling [1].

The governing equation of motion is the partial integro-differential equation

$$\rho(\mathbf{x})\partial_t^2\mathbf{u}(\mathbf{x},t) = \int_{\mathcal{H}(\mathbf{x})} \mathbf{f}(\mathbf{x},\hat{\mathbf{x}},\mathbf{u}(\mathbf{x},t),\mathbf{u}(\hat{\mathbf{x}},t),t)\,\mathrm{d}\hat{\mathbf{x}} + \mathbf{b}(\mathbf{x},t)\,, \quad \mathbf{x}\in\mathcal{V}\,,\ t>0\,,$$

for the displacement field $\mathbf{u} = \mathbf{u}(\mathbf{x},t)$ of a body that occupies the reference volume $\mathcal{V}$, supplemented by initial conditions for $\mathbf{u}(\cdot,0)$ and $\partial_t\mathbf{u}(\cdot,0)$. Here, ρ denotes the mass density, $\mathbf{f}$ the so-called pairwise force field that describes the interaction of material particles, and $\mathbf{b}$ collects outer forces. Moreover, $\mathcal{H}(\mathbf{x}) = \{\hat{\mathbf{x}}\in\mathcal{V} : \|\hat{\mathbf{x}}-\mathbf{x}\| \le \delta\}$ is the so-called peridynamic horizon for prescribed $\delta > 0$.

An essential feature is that $\mathbf{f}$ is independent of any spatial derivative. It is, therefore, a promising approach for the simulation of problems in which discontinuities emerge such as fracture or cracking.

The peridynamic modelling has recently been successfully applied in numerical simulations of problems in solid mechanics such as the fracture of a plate with notches, the undirected growth of cracks, the wrinkling and tearing of membranes, the deformation of composite materials etc.

In this talk, we give an overview of the peridynamic modelling as well as the numerical and theoretical results obtained so far. Concentrating on the description of a linear microelastic material, we present new results concerning the well-posedness of the problem. We then suggest a quadrature formula methods for the spatial approximation of the governing equation and show some numerical simulations. Moreover, the question of energy conservation and the comparison of elastic energy in both the peridynamic and the classical theory are discussed.

References

[1] S. Silling, Reformulation of elasticity theory for discontinuities and long-range forces. *J. Mech. Phys. Solids*, **48**, 175–209, 2000.