Recent Trends in Differential Equations: Analysis and Discretisation Methods

Technische Universität Berlin
November 7 - 9, 2013

Organisers:
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Moritz Kaßmann (Bielefeld)
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The workshop focusses on mathematical approaches to nonlinear and nonlocal phenomena by partial differential and integro-differential equations. The aim is to discuss the use of various methods and concepts such as fractional derivatives, truncation techniques, appropriate function spaces, and stochastic analysis. This workshop intends to bring together researchers from different mathematical schools and to offer an environment to foster the exchange of ideas.

We wish you all an inspiring workshop and a pleasant stay in Berlin.

Etienne Emmrich
Moritz Kaßmann
Petra Wittbold
Get Together

There will be an informal “get together” at Wohlfahrt’s Restaurant on Wednesday evening. The restaurant is located on the ground floor of the Hotel Indigo. We will meet at 7 pm and be there until 10:30 pm.

Program

Lectures will take place in room 313/314 which is located on the 3rd floor of the mathematics building (marked on the map on the last page of this booklet). Coffee Breaks take place in the adjoining room 315.

Conference Dinner

The conference dinner takes place at Neugrüns Köche, which is a small restaurant in one of the lively quarters of Berlin, on Thursday at 7 pm. Neugrüns Köche, Schönhauser Allee 135a, 10437 Berlin.

Transport

The metro line U2 connects the university at the station U Ernst-Reuter-Platz with the main points of interest of Berlin, where there are Zoologischer Garten, Potsdamer Platz and Alexanderplatz. Another possibility to get to Zoologischer Garten, Alexanderplatz and also to Hauptbahnhof is to take one of the city trains S5, S7 or S75 from the station S Tiergarten. In order to use the public transport you need to draw a ticket Berlin AB at a vending machine, which are accessible at each station. A single ticket (“Einzelfahrschein”) costs EUR 2,60. It could be smart to buy a daily ticket (“Tageskarte”) for EUR 6,70 or four single tickets at once (“4-Fahrten-Karte Einzelfahrschein”) for EUR 8,80. Make sure to validate your ticket at a stamp machine at the station before boarding.
Restaurants within Reach

Cafeteria Mathematics Building The Cafeteria at the entrance to the Mathematics Building. You can get snacks, sandwiches and lunch. Closed on Saturdays.

Staff Cafeteria Mathematics Building Not only for staff members, on the 9th floor, there is also a University Canteen, for many people the best place to have lunch in the university. Prices range from EUR 3,10 to 4,50. Closed on Saturdays.

Cafeteria Architecture Building Straße des 17. Juni 152, on the side of the Mathematics Building, you can get snacks, lunch, salads, self-made cakes. Prices range from EUR 2,00 to 4,50. Closed on Saturdays.

Cafeteria TU Skyline Ernst-Reuter-Platz 7 in the Telekom building. Beautiful view from the 20th floor, but few dishes and few seats. Closed on Saturdays.

Café Campus The Café Campus behind the blue-and-red Mathematics Building. The quiet atmosphere makes this the ideal place to meet in small groups between sessions.


Moon Thai 2 Knesebeckstr. 15. Thai cuisine with a great variety of dishes. Prices for main dishes from EUR 6,50 to 7,50. On Saturday not open until 5 pm.


Pratirio Knesebeckstr. 22. Typical Greek cuisine. Business-lunch from noon to 4 pm for EUR 7,90. Prices for main dishes from EUR 6,50.

Manjurani Knesebeckstr. 4. Indian Food. Prices for main dishes from EUR 5,00. Special lunch offers on chalk board outside.
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Abstracts

(sorted by the order of talks)

THURSDAY

Felix OTTO

Error estimates in stochastic homogenization

In many applications, one has to solve an elliptic equation with coefficients that vary on a length scale much smaller than the domain size. We are interested in a situation where the coefficients are characterized in statistical terms: Their statistics are assumed to be translation invariant and to decorrelate over large distances. As is known by qualitative theory, the solution operator behaves – on large scales – like the solution operator of an elliptic problem with \textit{homogeneous, deterministic} coefficients!

We are interested in several quantitative aspects: How close is the actual solution to the homogenized one – we give an optimal answer in terms of the quenched Green’s function, and point out the connections with elliptic regularity theory (input from Nash’s theory, a new outlook on De Giorgi’s theory).

We are also interested in the quantitative ergodicity properties for the process usually called “the environment as seen from the random walker”. We give an optimal estimate that relies on a link with (the Spectral Gap for) another stochastic process on the coefficient fields, namely heat-bath Glauber dynamics. This connection between statistical mechanics and stochastic homogenization has previously been used in opposite direction (i. e. with qualitative stochastic homogenization as an input).

Theory provides a formula for the homogenized coefficients, based on the construction of a “corrector”, which defines harmonic coordinates. This formula has to be approximated in practise, leading to a random and a systematic error. If time permits, we point out optimal estimates of both.

This is joint work with A. Gloria, S. Neukamm, and D. Marahrens.

Lisa BECK

Regularization by noise for the stochastic transport equation

In this talk several aspects of regularity and uniqueness are investigated for weak ($L^\infty$-) solutions to the (stochastic) transport equation

$$du = b \cdot Du \, dt + \sigma Du \circ dW_t.$$ 

Here, $b$ is a time dependent vector field (the drift), $u$ is the unknown, $\sigma$ is a real number, $W_t$ is a Brownian motion, and the stochastic term is interpreted in the Stratonovich sense.

For the deterministic equation ($\sigma = 0$) it is well-known that multiple solutions may exist and that solutions may blow up from smooth initial data in finite time if the drift is not regular enough (basically less than Lipschitz in space). For the stochastic
equation ($\sigma \neq 0$) instead, it turns out that a suitable integrability condition on the drift is sufficient to prevent the formation of non-uniqueness and singularities of the solutions.

In my talk I will explain this phenomenon of regularization by noise (which is a different mechanism than the presence of a viscosity term $\Delta u$); while a similar result was achieved recently via stochastic characteristics, we obtain the conservation of Sobolev regularity of the initial data with PDE techniques, under the aforementioned integrability condition. Starting from this regularity result (not available in the deterministic case), we can then apply a duality approach which in turn leads to path-by-path uniqueness.

The results presented in this talk are part of a joint project with F. Flandoli, M. Gubinelli and M. Maurelli.

**Rico ZACHER**

**Optimal decay estimates for non-local subdiffusion equations**

In my talk I will discuss recent results on the temporal decay of solutions to non-local in time subdiffusion problems like

$$\partial_t \int_0^t k(t-\tau) (u(\tau,x) - u_0(x)) d\tau - \text{div} (A(t,x)\nabla u) = 0, \quad t > 0, \ x \in \Omega,$$

where $k$ is a positive and non-increasing kernel that is singular at $t = 0$, and $A$ is bounded and measurable and satisfies a uniform parabolicity condition. An important example is given by the time-fractional diffusion equation of time order $\alpha \in (0, 1)$. I will consider both the case of a bounded domain $\Omega$ with homogeneous Dirichlet condition and the full space case $\Omega = \mathbb{R}^d$. In the case $\Omega = \mathbb{R}^d$ the focus lies on the special case $A = I d$.

The goal is to obtain optimal decay estimates for $u(t)$ in $L^p(\Omega)$. It turns out that there are some fundamental and interesting differences to the classical heat equation. I will further comment on corresponding results for the time-fractional $p$-Laplace equation.

This is partially joint work with Vicente Vergara (Tarapaca, Chile), Jukka Kempo-painen (Oulu, Finland), and Juhana Siljander (Helsinki, Finland).

**François MURAT**

**Existence and homogenization of the solution of the problem**

$$-\text{div} A(x) Du_\varepsilon = \frac{f(x)}{u_\varepsilon} + g(x) \quad \text{in } \Omega_\varepsilon, \quad u = 0 \text{ on } \partial \Omega_\varepsilon,$$

when $\Omega_\varepsilon$ is a perforated domain “à la Cioranescu-Murat”

In this recent joint work with Daniela Giachetti (Università di Roma La Sapienza) and Pedro J. Martínez-Aparicio (Universidad Politécnica de Cartagena), we consider a nonnegative solution of the homogeneous Dirichlet problem obtained by the perturbation of a second order linear equation by a nonnegative term which is singular when the solution vanishes (see the title for the prototype of this problem).
We first introduce an “ad hoc” definition of the solution of this problem. For any given open set $\Omega_\varepsilon$, we then prove the existence, uniqueness, and stability of the solution defined in this sense. We finally perform the homogenization of this singular homogeneous Dirichlet problem posed on a sequence of domains with many small holes “à la Cioranescu-Murat” in which a “strange term” of zeroth order appears at the limit.

**Russell SCHWAB**

**Integro-differential methods for Neumann homogenization**

We use a recent result about the representation of the Dirichlet-to-Neumann operator for fully nonlinear equations as an integro-differential operator on the boundary of the domain to guide the analysis of the homogenization problem with oscillatory Neumann data.

This allows to use methods already established for integro-differential equations. We will present the case of a periodic environment with a half-space domain whose boundary is an irrationally oriented hyperplane, and this results in the study of a global almost periodic nonlocal equation on the hyperplane.

This is a joint work with Nestor Guillen.

**Erik LINDGREN**

**Pointwise estimates for parabolic equations and the parabolic obstacle problem**

I will discuss the pointwise regularity of solutions to parabolic equations. In particular, I will talk about a method that can be used to obtain a second order Taylor expansion (in $L^p$-average). I will also discuss how the same method can be applied to the parabolic obstacle problem in order to study the regularity of the free boundary.

**Giuseppe Rosario MINGIONE**

**Update on nonlinear potential theory**

The classical linear potential theory describes estimates and fine properties for solutions to linear elliptic and parabolic equations via linear Riesz potentials. Nonlinear potential theory deals with similar issues, but when the equations are nonlinear, and possibly degenerate. The p-Laplacean operator is a basic example here. I will try to give a survey of recent results in this setting.

**Cyril IMBERT**

**The vertex test function for Hamilton–Jacobi equations on networks**

A general method for proving comparison principles for Hamilton–Jacobi equations on networks is introduced. It consists in constructing a vertex test function to be used in the doubling variable technique. The first important consequence is that it provides very general existence and uniqueness results for Hamilton–Jacobi equations
on networks with Hamiltonians that are not convex with respect to the gradient variable and can be discontinuous with respect to the space variable at vertices. It also opens many perspectives for the study of these equations in such a singular geometrical framework; to illustrate this fact, we show how to derive a homogenization result for networks from the comparison principle.

FRIDAY

Tadele MENGESHA
Multiscale analysis of a linear peridynamic solid

I will present a recent work on the homogenization of the state-based heterogeneous peridynamic model. The model involves a multiscale nonlocal interaction in the form of long range forces with highly oscillatory perturbations that represent the presence of heterogeneity on a smaller spatial length scale.

The two-scale convergence theory is established for the multiscale linear steady state variational problem as well as the peridynamic equation of motion. We also present some regularity results for the homogenized nonlocal equation and present a strong approximation to the solution of the peridynamic model via a scaling of the solution of two-scale homogenized equation.

This is a joint work with Q. Du and R. Lipton.

Robert LIPTON
Dynamic PDE based fracture propagation as a distinguished limit of unstable nonlocal bond models

The talk addresses nonlocal dynamic models for solving problems of free crack propagation described by the Peridynamic formulation. Here we investigate distinguished small horizon limits of Peridynamic evolutions associated with unstable bond models. The associated Peridynamic evolutions are shown to converge to a PDE based dynamic brittle fracture evolution described by an evolving crack set together with the linear elastic wave equation for the displacement off the crack set. The limit evolution has bounded energy expressed in terms of the bulk and surface energies of brittle fracture mechanics. In this way it is seen that Peridynamic evolutions converge to free crack propagation as described by PDE based models.
What is between conservative and dissipative solutions for the Camassa–Holm equation?

The Camassa–Holm (CH) equation reads

\[ u_t - u_{txx} + \kappa u_x + 3uu_x - 2u_x u_{xx} - uu_{xxx} = 0 \]

where \( \kappa \) is a real parameter. We are interested in the Cauchy problem on the line with initial data in \( H^1 \). There is a well-known and well-studied dichotomy between two distinct classes of solutions of the CH equation. The two classes appear exactly at wave breaking where the spatial derivative of the solution becomes unbounded while its \( H^1 \) norm remains finite. We here introduce a novel solution concept gauged by a continuous parameter \( \alpha \) in such a way that \( \alpha = 0 \) corresponds to conservative solutions and \( \alpha = 1 \) gives the dissipative solutions. This allows for a detailed study of the difference between the two classes of solutions and their behavior at wave breaking. We also extend the analysis to a two-component Camassa–Holm system. This is joint work with Katrin Grunert (NTNU) and Xavier Raynaud (SINTEF).

Existence of global weak solutions to implicitly constituted kinetic models of incompressible homogeneous dilute polymers

The talk will survey recent joint work with Miroslav Bulíček and Josef Málek at the Mathematical Institute, Faculty of Mathematics and Physics, Charles University, Prague.

We show the existence of global weak solutions to a general class of kinetic models of homogeneous incompressible dilute polymers. The main new feature of the model is the presence of a general implicit constitutive equation relating the viscous part \( S_v \) of the Cauchy stress and the symmetric part \( D \) of the velocity gradient. We consider implicit relations that generate maximal monotone (possibly multivalued) graphs, and the corresponding rate of dissipation is characterized by the sum of a Young function and its conjugate depending on \( D \) and \( S_v \), respectively. Such a framework is very general and includes, among others, classical power-law fluids, stress power-law fluids, fluids with activation criteria of Bingham or Herschel–Bulkley type, and shear-rate dependent fluids with discontinuous viscosities as special cases. The appearance of \( S_v \) and \( D \) in all the assumptions characterizing the implicit relationship \( G(S_v, D) = 0 \) is fully symmetric. The elastic properties of the flow, characterizing the response of polymer macromolecules in the viscous solvent, are modelled by the elastic part \( S_e \) of the Cauchy stress tensor, whose divergence appears on the right-hand side of the momentum equation, and which is defined by the Kramers expression involving the probability density function, associated with the random motion of the polymer molecules in the solvent. The probability density function satisfies a Fokker–Planck equation, which is nonlinearly coupled to the momentum equation.
We establish long-time and large-data existence of weak solutions to such a system, completed by an initial condition and either a no-slip or Navier’s slip boundary condition, by using properties of maximal monotone operators and Lipschitz approximations of Sobolev-space-valued Bochner functions via a weak compactness argument based on the Div-Curl Lemma and Chacon’s Biting Lemma. A key ingredient in the proof is the strong compactness in $L^1$ of the sequence of Galerkin approximations to the probability density function and of the associated sequence of approximations to the elastic part $S_{\varepsilon}$ of the Cauchy stress tensor.

Bibliography


Andreas PROHL

Optimal control in evolutionary micromagnetism

We consider an optimal control problem subject to the Landau–Lifshitz–Gilbert equation (LLG)

$$m_t = -\alpha m \times (m \times \Delta m) + m \times (\Delta m + u)$$

which describes the evolution of magnetizations $m$ in $S^2$. Here $u : [0, T] \times \Omega \to \mathbb{R}^3$ is an applied field which is optimized according to some quadratic functional. The problem is motivated in order to control switching processes in ferromagnets. I start with a survey of existing numerical schemes which approximate solutions of LLG. A main focus here is to properly discretize the sphere property of solutions. Then, I discuss the optimality system for the optimal control problem, and a semi-discretization of it. I discuss convergence of the latter method. Computational studies will be shown.

This is joint work with T. Dunst, M. Klein, and A. Schäfer (U Tübingen).

Lars DIENING

Instance optimality for the maximum strategy

We study the adaptive finite element approximation of the Dirichlet problem $-\Delta u = f$ with zero boundary values using linear Ansatz functions and newest vertex bisection. Our approach is based on the minimization of the corresponding Dirichlet energy. We show that the maximum strategy attains every energy level with a number of degrees of freedom, which is proportional to the optimal number. As a consequence we achieve instance optimality of the error.

This is a joint work with Christian Kreuzer (Bochum) and Rob Stevenson (Amsterdam).
Guy VALLET

The Dirichlet problem for a conservation law with a multiplicative stochastic force

In this talk, we are interested in the Dirichlet problem:

\[ du = \text{div} \vec{f}(u) \, dt + g(x, u) \, dt + h(x, u) \, dW \text{ on } \Omega \times (0, T) \times D, \]

with the formal condition \( u = 0 \) on \( \partial D \) and the initial condition \( u(t = 0) = u_0 \).

One assumes that

- \( D \subset \mathbb{R}^d \) bounded and Lipschitz and \( u_0 \in L^2(D) \).
- \( \vec{f} : \mathbb{R} \to \mathbb{R}^d \) is Lipschitz-continuous and \( \vec{f}(0) = \vec{0} \).
- \( h, g : \mathbb{R}^{d+1} \to \mathbb{R} \) are uniformly Lipschitz-continuous in \( u \) and \( h(\cdot, 0) = 0 \).
- \( h \) is uniformly \( \alpha \)-Hölder-continuous in \( x \) with \( \alpha > 1/2 \).
- \( W = \{ W_t, \mathcal{F}_t, 0 \leq t \leq T \} \) is a real adapted continuous Brownian motion on the classical Wiener space \((\Omega, \mathcal{F}, P)\) for the filtration \((\mathcal{F}_t)\).

After giving the definition of a weak entropy solution, we present a method to prove the existence and the uniqueness of such a solution.

Then, we propose some numerical simulations.

Bibliography

Boris ANDREIANOV

On boundary-value problems for hyperbolic and degenerate parabolic
conservation laws

The talk is devoted to the problems that can be written under the form

\[ u_t + \text{div} \left( f(u) - \nabla \phi(u) \right) = 0 \quad \text{in} \quad (0, T) \times \Omega \]

with initial condition and different boundary conditions (BC). This setting contains both pure hyperbolic conservation laws ($\phi \equiv 0$) and strongly degenerate convection-diffusion equations encountered in some porous medium, sedimentation and road traffic models. So far, only the Dirichlet BC $u = u_D$ on $(0, T) \times \partial \Omega$ has been studied extensively, starting from the celebrated work [3].

Our main goal is to develop general approaches for treating, in a unified way, different boundary conditions: Dirichlet, zero-flux, obstacle,... We are also interested in efficient numerical approximation of such problems.

Because of purely hyperbolic degeneracy, as in the case of [3] the appropriate formulation or the BC should be seen as a singular limit formulation. One can start with a numerical scheme or a vanishing viscosity approximation, where the BC are taken into account in a straightforward way, and derive the formulation satisfied at the limit. At the limit, it may happen that the BC undergo a projection procedure: namely, the limit satisfies an effective BC that can differ from the formal BC prescribed for the approximations. We attempt to describe this procedure. This goal is achieved in the pure hyperbolic setting in [2].

In the degenerate parabolic setting, the zero-flux BC $(f(u) - \nabla \phi(u)) \cdot \nu = 0$ was treated in [1]. This practically important BC will receive a special attention.

Bibliography


Agnes LAMACZ

Homogenization of waves in periodic media: Long time behavior and dispersive effective equations

We study second order linear wave equations in periodic media with a small periodicity length $\varepsilon > 0$,

$$\partial_t^2 u^\varepsilon(x, t) = \nabla \cdot \left( a \left( \frac{x}{\varepsilon} \right) \nabla u^\varepsilon(x, t) \right),$$

(1)

aiming at the derivation of effective equations in $\mathbb{R}^n$. Standard homogenization theory provides, for the limit $\varepsilon \to 0$, an effective second order wave equation that describes solutions of (1) on time intervals $[0, T]$. In this talk a refinement of this classical result is presented: We investigate the behavior of solutions $u^\varepsilon$ on large time intervals $[0, T\varepsilon^{-2}]$ and show that in order to approximate the solutions for all $t \in [0, T\varepsilon^{-2}]$ one has to use a dispersive, higher order wave equation. We provide a well-posed fourth order effective constant coefficient equation of the form

$$\partial_t^2 w^\varepsilon = AD^2 w^\varepsilon + \varepsilon^2 ED^2 \partial_t^2 w^\varepsilon - \varepsilon^2 FD^4 w^\varepsilon$$

(2)

and estimate the errors between the solution $u^\varepsilon$ of the original heterogeneous problem and the solution $w^\varepsilon$ of the dispersive wave equation. We demonstrate that the main difficulty lies in the derivation of (2), which is performed in two steps. In the first step we identify a family of relevant, not necessarily well-posed limit models via classical Bloch wave analysis and present also an alternative approach: adaption operators. In the second step a method to select the well-posed effective model (2) is provided.

Mark STEINHAUER

On Liouville-type theorems for (steady) Navier–Stokes equations respectively a class of nonlinear elliptic systems

We study Liouville-type properties for solutions to the steady incompressible Navier–Stokes equations in $\mathbb{R}^3$ and show that a solution $v$ satisfying $v(x) \to 0$ as $|x| \to +\infty$ and $\int_{\mathbb{R}^3} |\nabla v|^2 \, dx < +\infty$ is equal to zero. We introduce and discuss a corresponding problem for solutions of a class of nonlinear elliptic systems.

Benjamin GESS

Finite time extinction for stochastic sign fast diffusion and self-organized criticality

We will first shortly review the informal derivation of a continuum limit for the Bak–Tang–Wiesenfeld model of self-organized criticality. This will lead to the stochastic sign fast diffusion equation. A key property of models exhibiting self-organized criticality is the relaxation of supercritical states into critical ones in finite time. However, it has remained an open question for several years whether the continuum limit – the stochastic sign fast diffusion – satisfies this relaxation in finite time. We will present a proof of this.
Carsten CARSTENSEN

Crouzeix–Raviart nonconforming FEM are more useful than formerly expected

The presentation discusses the status of nonconforming finite element methods in the textbooks and the medius analysis and known comparison results. This establishes that the nonconforming discretisations are not worse than the conforming ones. The surprising advantages for nonlinear problems conclude the presentation with applications to guaranteed lower bounds of eigenvalues and energies.

Alexander MIELKE

On the geometry of reaction-diffusion systems: Optimal transport versus reaction

General reaction-diffusion systems satisfying a detailed-balance condition can be formulated in a natural way as a gradient system for the relative entropy. The geometric gradient is determined via an Onsager operator containing a diffusion part of Wasserstein type and a reaction term.

We discuss how the recently developed theory for optimal transport can be generalized to such cases giving results on exponential decay and geodesic convexity. In particular we discuss an example where the Wasserstein distance is replaced by the Hellinger–Kantorivich distance.

This is joint work with Matthias Liero and Giuseppe Savaré.
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<td>09:30–10:15</td>
<td>Error estimates in stochastic homogenization Felix Otto</td>
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<td>On boundary-value problems for hyperbolic and degenerate parabolic conservation laws Boris Andreianov</td>
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<td>10:15–11:00</td>
<td>Regularization by noise for the stochastic transport equation Lisa Beck</td>
<td>Multiscale analysis of a linear peridynamic solid Tadele Mengesha</td>
<td>Homogenization of waves in periodic media: Long time behavior and dispersive effective equations Agnes Lamacz</td>
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<td>11:00–11:30</td>
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<td>11:30–12:15</td>
<td>Optimal decay estimates for non-local subdiffusion equations Rico Zacher</td>
<td>Dynamic PDE based fracture propagation as a distinguished limit of unstable nonlocal bond models Robert Lipton</td>
<td>On Liouville-type theorems for (steady) Navier–Stokes equations respectively a class of nonlinear elliptic systems Mark Steinrueger</td>
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<td>12:15–13:00</td>
<td>Existence and homogenization when $\Omega_\varepsilon$ is a perforated domain “à la Cioranescu-Murat” François Murat</td>
<td>What is between conservative and dissipative solutions for the Camassa–Holm equation? Helge Holden</td>
<td>Finite time extinction for stochastic sign fast diffusion and self-organized criticality Benjamin Gess</td>
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<td>14:30–15:15</td>
<td>Integro-differential methods for Neumann homogenization Russell Schwab</td>
<td>Existence of global weak solutions to implicitly constituted kinetic models of incompressible homogeneous dilute polymers Endre Süli</td>
<td>Crouzeix–Raviart nonconforming FEM are more useful than formerly expected Carsten Carstensen</td>
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<td>15:15–16:00</td>
<td>Pointwise estimates for parabolic equations and the parabolic obstacle problem Erik Lindgren</td>
<td>Optimal control in evolutionary micromagnetism Andreas Prohl</td>
<td>On the geometry of reaction-diffusion systems: Optimal transport versus reaction Alexander Mielke</td>
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<td>Update on nonlinear potential theory Giuseppe Rosario Mingione</td>
<td>Instance optimality for the maximum strategy Lars Diening</td>
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<td>17:15–18:00</td>
<td>The vertex test function for Hamilton–Jacobi equations on networks Cyril Imbert</td>
<td>The Dirichlet problem for a conservation law with a multiplicative stochastic force Guy Vallet</td>
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<td>19:00</td>
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