Existence of global weak solutions to implicitly constituted kinetic models of incompressible homogeneous dilute polymers

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Abstract

The talk will survey recent joint work with Miroslav Bulíček and Josef Málek at the Mathematical Institute, Faculty of Mathematics and Physics, Charles University, Prague.

We show the existence of global weak solutions to a general class of kinetic models of homogeneous incompressible dilute polymers. The main new feature of the model is the presence of a general implicit constitutive equation relating the viscous part $\mathbf{S}_v$ of the Cauchy stress and the symmetric part $\mathbf{D}$ of the velocity gradient. We consider implicit relations that generate maximal monotone (possibly multivalued) graphs, and the corresponding rate of dissipation is characterized by the sum of a Young function and its conjugate depending on $\mathbf{D}$ and $\mathbf{S}_v$, respectively. Such a framework is very general and includes, among others, classical power-law fluids, stress power-law fluids, fluids with activation criteria of Bingham or Herschel–Bulkley type, and shear-rate dependent fluids with discontinuous viscosities as special cases. The appearance of $\mathbf{S}_v$ and $\mathbf{D}$ in all the assumptions characterizing the implicit relationship $\mathbf{G}(\mathbf{S}_v, \mathbf{D}) = \underline{0}$ is fully symmetric. The elastic properties of the flow, characterizing the response of polymer macromolecules in the viscous solvent, are modelled by the elastic part $\mathbf{S}_e$ of the Cauchy stress tensor, whose divergence appears on the right-hand side of the momentum equation, and which is defined by the Kramers expression involving the probability density function, associated with the random motion of the polymer molecules in the solvent. The probability density function satisfies a Fokker–Planck equation, which is nonlinearly coupled to the momentum equation.

We establish long-time and large-data existence of weak solutions to such a system, completed by an initial condition and either a no-slip or Navier's slip boundary condition, by using properties of maximal monotone operators and Lipschitz approximations of Sobolev-space-valued Bochner functions via a weak compactness argument based on the Div-Curl Lemma and Chacon's Biting Lemma. A key ingredient in the proof is the strong compactness in L^1 of the sequence of Galerkin approximations to the probability density function and of the associated sequence of approximations to the elastic part $\mathbf{S}_e$ of the Cauchy stress tensor.

Bibliography

M. Bulíček, J. Málek, and E. Süli. Existence of global weak solutions to implicitly constituted kinetic models of incompressible homogeneous dilute polymers. *Comm. Partial Differential Equations*, Volume 38, Issue 5 (2013), 882–924.