The Dirichlet problem for a conservation laws with a multiplicative stochastic force

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In this talk, we are interested in the Dirichlet problem:

$$\mathrm{d}u = \operatorname{div} \vec{\mathbf{f}}(u)\,\mathrm{d}t + g(x,u)dt + h(x,u)\,\mathrm{d}W \text{ on } \Omega \times (0,T) \times \mathrm{D},$$

with the formal condition $u = 0$ on $\partial\mathrm{D}$ and the initial condition $u(t=0) = u_0$.

One assumes that

- $\mathrm{D} \subset \mathbb{R}^d$ bounded and Lipschitz and $u_0 \in L^2(\mathrm{D})$.
- $\vec{\mathbf{f}} : \mathbb{R} \to \mathbb{R}^d$ is Lipschitz-continuous and $\vec{\mathbf{f}}(0) = \vec{0}$.
- $h, g : \mathbb{R}^{d+1} \to \mathbb{R}$ are uniformly Lipschitz-continuous in u and $h(\cdot, 0) = 0$.
- h is uniformly α-Hölder-continuous in x with $\alpha > 1/2$.
- $W = \{W_t, \mathcal{F}_t, 0 \le t \le T\}$ is a real adapted continuous Brownian motion on the classical Wiener space $(\Omega, \mathcal{F}, P)$ for the filtration $(\mathcal{F}_t)$.

After giving the definition of a weak entropy solution, we present a method to prove the existence and the uniqueness of such a solution.

Then, we propose some numerical simulations.

References

[1] C. Bauzet, G. Vallet, and P. Wittbold. The Dirichlet problem for a conservation law with a multiplicative stochastic perturbation. To appear in J. Funct. Anal. dx.doi.org/10.1016/j.jfa.2013.06.022.