Time-fractional stochastic conservation laws

(joint work with Martin Scholtes)

We consider time-fractional scalar conservation laws of the form

$$dg_{1-\alpha} * (u - u_0) + \text{ div } f(u)dt = I^{1-\beta} hdW,$$

where $g_{1-\alpha}(t) = \frac{t^{-\alpha}}{\Gamma(1-\alpha)}$ $(\alpha \in (0,1))$, i.e.,

$$\partial_t g_{1-\alpha} * (u - u_0) = \partial_t^\alpha (u - u_0)$$

is the fractional time-derivative in the sense of Riemann-Liouville,

$$f : \mathbb{R} \to \mathbb{R}^N \text{ is a smooth function, and}$$

$I^{1-\beta}$ is the fractional integral of order $1-\beta$ in the sense of Riemann-Liouville ($\beta = 1$ corresponds to the classical additive stochastic noise hdW with a given function h and $W = (W(t), \mathcal{F}_t, 0 \leq t \leq T)$ one-dimensional Brownian motion on a classical Wiener space).

Under certain assumptions on α and β we prove existence and uniqueness of stochastic entropy solutions for arbitrary L^2-initial data.

An interesting open question is whether it is possible to generalize these results to the case of a multiplicative stochastic noise. The main difficulty is that an Itô type formula is not known to exist in the time-fractional derivative case.