Dr. Philipp J. di Dio – Curriculum Vitae

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Research Interests

- Moment Problem (truncated and full; measurable; real and complex; algebraic; differential structure; applications in optimization, (algebraic) statistics, and shape reconstruction and recognition; signal processing)
- Partial Differential Equations

Summary of Scientific Achievements

My main research field is the (truncated) moment problem and its applications. Starting with my PhD-Thesis under supervision of Konrad Schmüdgen we investigated the multi-dimensional moment problem, especially the truncated case. Since every truncated moment problem has a finitely atomic representing measure (Richter Theorem) we were interested in the Carathéodory number and found several lower and upper bounds. The most general lower bound $\left\lceil \frac{m}{n+1} \right\rceil$, m number of moments and n dimension of the underlying space, based on Sard's Theorem not only improved all existing lower bounds but also stays the best general lower bound. Additionally, existence and determinacy have been investigated and significant results have been obtained, see [7-9].
Moving forward I looked at mixtures $g = \sum_{i=1}^{k} c_i \cdot g_i$, $c_i > 0$, of Gaussian, log-normal, and more general distributions g_i and their moments $\int x^\alpha \cdot g(x) \, \mathrm{d}^n x$ [10]. Previous attempts concentrated on the algebraic side of the Gaussian distributions. However, in [10] I was able to completely characterize the moment sequences which have a mixture representation and I gave the first non-trivial bounds on the number k of components for any given g_i's. This was achieved not by looking algebraically but analytically at the problem and has the two advantages that, firstly, the properties enforcing existence and number of components were singled out (polynomials have to much structure which can not be separated but general measurable or continuous functions are much more flexible to determine the cause), and secondly the results are now so general that all cases are covered and further generalization is very hard or impossible.
Joining Mario Kummer in Berlin we worked deeper at the algebraic side of the moment problem. We significantly improved the lower bounds on the Carathéodory number for the cases of polynomials using Hilbert functions. We found that already atomic measures supported on a grid $\{1, 2, \ldots, d\}^n$ provide examples of moment functionals $L : \mathbb{R}[x_1, \ldots, x_n]_{\leq 2d} \to \mathbb{R}$ which need exactly

$$\binom{n+2d}{n} - n \cdot \binom{n+d}{n} + \binom{n}{2} \tag{1}$$

atoms and therefore we have the asymptotic results that for any $d \in \mathbb{N}$ and $\varepsilon > 0$ there is an $n \in \mathbb{N}$ such that the Carathéodory number is bounded from below by $(1-\varepsilon) \cdot \binom{n+2d}{n}$ where $\binom{n+2d}{n}$ is the number of moments, see [P3]. This also answered the long standing question to which degree the flat extension must be calculated: The worst case is attained already at low dimensions and degrees, see [P3] for more details.
Since from [10] still important questions about the number of components and the reconstruction of representing measure were open I looked in [P4] deeper in this matter. Starting with the reconstruction of polytopes from moments I developed a new technique of derivatives of moments resp. moment functionals:

$$\partial^\alpha L := (-1)^{|\alpha|} \cdot L \circ \partial^\alpha.$$

I formally introduces and investigated this concept. I was then able to prove several reconstruction results of measures (i.e., functions and shapes) in a unified and concise way and was able to extend them from

the polynomial case to the measurable case and more complicated shapes. E.g. when L is represented by $f(x) = \exp(c_0 + c_1 x + \cdots + c_n x^n)$ with $c_i \in \mathbb{R}$, then

$$\partial L = c_1 L + 2c_2 ML + \cdots + nc_n M^n L$$

where M is the shift of the moment functional, see [P4] for more details. Going to mixtures of Gaussians in dimension $n = 1$ I was able to provide a method/algorithm to determine the coefficients and variances of the mixture by linear algebra, i.e., I reduced the highly non-trivial fitting to the well-studied problem of finding all roots of one univariable polynomial of degree n. In higher dimensions I characterized all moment sequences which are represented by one Gaussian. For mixtures of Gaussians in higher dimensions the required number of components was investigated with the new results from [P3]. While this was a long asked question which was treated with algebraic methods, i.e., solving huge polynomial systems, I was able to show that the lower bound in eq. (1) also holds for Gaussian mixtures.
Working on the derivatives of moments I got more interested in partial differential equations and the possible application of the moment problem to such problems. As a toy example I looked at

$$\partial_t f(x,t) = \nu \cdot \Delta f(x,t) + g(x,t) \cdot \nabla f(x,t) + h(x,t) \cdot f(x,t) + k(x,t) \tag{2}$$

with $f(x,0) = f_0(x) \in \mathcal{S}(\mathbb{R}^n)^m$ Schwartz function initial data, where $g = (g_1, \ldots, g_n)^t$, $h = (h_{ij})_{i,j=1}^m$, $k = (k_1, \ldots, k_m)^t$, and $g_i, h_{ij} \in C_b^\infty = \{f \in C^\infty(\mathbb{R}^n) \,|\, \|\partial^\alpha f\|_\infty < \infty \text{ for all } \alpha \in \mathbb{N}_0^n\}$, $k_i \in \mathcal{S}(\mathbb{R}^n)$. Besides existence of a solution I was able to prove $f(\,\cdot\,, t) \in \mathcal{S}(\mathbb{R}^n)^m$ for all $t \geq 0$ and I found explicit bounds on the semi-norms

$$p_{\alpha,\beta}(f(\,\cdot\,,t)) := \left\| x^\alpha \cdot \partial^\beta f(x,t) \right\|_\infty \leq C_{\alpha,\beta}(t). \tag{3}$$

This result provided a new existence and smoothness criteria for the Euler and the Navier–Stokes equations resp. its vorticity formulation, see [P5] for more details.

Education

05/2018 Dissertation in Mathematics - University of Leipzig, Leipzig, Germany

Degree	Dr. rer. nat. (PhD, magna cum laude)
Thesis	*The Truncated Moment Problem*
First Referee	Prof. Dr. Konrad Schmüdgen
Second Referee	Prof. Dr. Claus Scheiderer

03/2014–02/2018 PhD Studies - Universtity of Leipzig, Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany

Supervisor	Prof. Dr. Konrad Schmüdgen

10/2008–09/2013 Diploma in Mathematics - University of Leipzig, Leipzig, Germany

Degree	Dipl.-Math. (very good, 1.2)
Thesis	*Discrete Crossed Products and Discrete Partial Crossed Products* (very good, 1.0)
Supervisor	Prof. Dr. Konrad Schmüdgen

04/2007–03/2008 Absence due to Illness

10/2005–03/2007 Bachelor of Science in Chemistry - University of Leipzig, Leipzig, Germany

04/2008–09/2009

Degree	B.Sc. (good, 1.7)
Thesis	*Theoretical Studies of the Rutheniumtetroxide catalyzed oxidative Cyclization of 1,5- and 1,6-Dienes* (very good, 1.0)
Supervisor	Prof. Dr. Barbara Kirchner
Special Projects	Measurements at the BESSY synchrotron facility Berlin, 19th to 31st March 2007, Supervisor: Dr. Igor Konovalov

07/2005 High School - Martin Luther Gymnasium, Lutherstadt Wittenberg, Germany

Degree	Abitur (good, 1.7)
Award	German Physical Society DPG Book Price 2005

Employment History

10/2018 – Present Technical University Berlin, Department for Mathematics, Berlin, Germany
Post Doc, Computer Algebra Group, Prof. Dr. Mario Kummer
- Real algebraic geometry for mathematicians
- Mathematics for engineers and physicists

04/2019 – 09/2019 University of Greifswald, Department of Mathematics, Greifswald, Germany
Lecturer Replacement, Prof. Dr. Mareike Fischer
- Mathematics II (calculus) for biochemists and environmental scientists including exercise and exam organization

03/2014 – 09/2018 University of Leipzig, Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany
PhD student and Post Doc, Prof. Dr. Konrad Schmüdgen
- Calculus and linear algebra for mathematicians, physicists, and teachers
- PhD grant, Max Planck Institute for Mathematics in the Sciences
- PhD/research grant, Deutsche Forschungsgemeinschaft (DFG), SCHM1009/6-1, Prof. Konrad Schmüdgen

04/2012 – 02/2014 University of Leipzig, Department for Mathematics and Computer Science, Leipzig, Germany
(Student) Teaching Assistant, Institute for Mathematics
- Calculus and linear algebra for mathematicians
- Mathematics for physicists and teachers

07/2009 – 12/2011 University of Leipzig, Department for Chemistry and Mineralogy, Leipzig Germany
Student Research Assistant, Institute for Theoretical Chemistry, Prof. Dr. Barbara Kirchner
- Static quantum chemical calculations with Turbomole
- Ab initio molecular dynamics simulations with cp2k
- Analysis of molecular dynamics simulations with Travis

Conference and Invited Talks

4th March 2020 Invited Talk, Goethe University, Frankfurt, Germany, Prof. Dr. Thorsten Theobald: *Shape reconstruction from Moments*

26th July 2019 International Workshop on Operator Theory and Applications (IWOTA) 2019, Lisbon, Portugal: *Carathéodory Numbers and Reconstruction of Measures from Moments*

10th May 2019 Invited Talk, University of Konstanz, Konstanz, Germany, Prof. Dr. Markus Schweighofer: *Carathéodory Numbers, Flat Extension, Evaluation Polynomials - The truncated Moment Problem*

2nd April 2019 A^3 Arctic Applied Algebra, Artic University of Tromso, Tromso, Norway: *The truncated Moment Problem: Recent Advances in Carathéodory Numbers*

26th Sept 2018 Summer School in Algebraic Statistics, Artic University of Norway, Tromso, Norway: *The Truncated Moment Problem applied to Statistics: Gaussian and Log-Normal Mixtures*

10th June 2017 Invited Talk, Work shop Polynomials and Polytopes, Prof. B. Sturmfels and Dr. T. de Wolff, Technical University Berlin, Berlin, Germany: *The Multi-Dimensional Truncated Moment Problem - Connections to Polytopes, Polynomials, and Waring Rank*

Conferences, Summer/Winter Schools, and Workshops (Selection)

27th – 28th Febr 2020 Milestone Conference Algebraic Geometry, Zuse Institute Berlin, Berlin, Germany

5th – 8th Aug 2019 6th International Conference on Continuous Optimization, Technical University Berlin, Berlin, Germany

22nd – 26th July 2019 30th International Workshop on Operator Theory and its Applications (IWOTA) 2019, Instituto Superior Técnica, Lisbon, Portugal

1st – 5th April 2019	A^3 - Arctic Applied Algebra, Arctic University of Norway, Tromso, Norway
24th – 28th Sept 2018	Summer School in Algebraic Statistics, Arctic University of Norway, Tromso, Norway
21st – 25th Aug 2017	Sums of Squares - Real Algebraic Geometry and its Applications, University of Innsbruck, Austria
10th June 2017	Work shop Polynomials and Polytopes, Technical University of Berlin, Berlin, Germany
5th – 11th March 2017	Real Algebraic Geometry with a View toward Moment Problems and Optimization, Mathematisches Forschungsinstitut Oberwolfach, Oberwolfach, Germany
11th – 15th July 2016	Summer School on Real Algebraic Geometry and Optimization, Georgia Tech, Atlanta GA, USA
7th – 9th April 2016	Andrejewski-Tage, University of Konstanz, Konstanz, Germany
12th – 16th Oct 2015	Ordered Algebraic Structures and Related Topics, Centre International de Rencontres Mathématique (CIRM), Marseille, France
29th June – 7th July 2015	BMS Summer School: Convex Geometry - Discrete and Computational, Technical University Berlin, Berlin, Germany

Publications (Peer Reviewed)

[10] P. J. di Dio: *The multidimensional truncated Moment Problem: Gaussian and Log-Normal Mixtures, their Carathéodory Numbers, and Set of Atoms*, Proc. Amer. Math. Soc. **2019**, 147, 3021–3038

[9] P. J. di Dio, K. Schmüdgen: *The multidimensional truncated Moment Problem: Atoms, Determinacy, and Core Variety*, J. Funct. Anal. **2018**, 274, 3124–3148

[8] P. J. di Dio, K. Schmüdgen: *The multidimensional truncated Moment Problem: Carathéodory Numbers*, J. Math. Anal. Appl. **2018**, 461, 1606–1638

[7] K. Schmüdgen, P. J. di Dio: *Truncated Moment Problem: Set of Atoms and Carathéodory Numbers*, Oberwolfach Reports **2017**, 14, 77–79

[6] P. J. di Dio: *Thermal Stability of Water up to Super-Critical States: Application of the Singular Value Decomposition and Grund Functions*, J. Mol. Liq. **2013**, 187, 207–217

[5] B. Kirchner, P. J. di Dio, J. Hutter: *Real-World Predictions from Ab Initio Molecular Dynamics Simulations*, Top. Curr. Chem. **2012**, 307, 109–154

[4] P. J. di Dio, M. Brüssel, K. Muniz, R. S. Ray, S. Zahn, B. Kirchner: *Pd-N to Pd-O Rearrangement for a Carbamate Synthesis from Carbon Dioxide and Methane: A Density Functional and Ab Initio Molecular Dynamics Metadynamics Study*, Angew. Chemie (DFG Special Edition, DOI:10.002/anie.201105813), **2011**, A40–A45

[3] P. J. di Dio, M. Brehm, B. Kirchner: *Singular Value Decomposition for Analyzing Temperature- and Pressure-Dependent Radial Distribution Functions: Decomposition into Grund RDFs (GRDFs)*, J. Chem. Theory Comput. **2011**, 7(10), 3035–3040

[2] M. Brüssel, P. J. di Dio, K. Muniz, B. Kirchner: *Comparison of Free Energy Surfaces Calculations from Ab Initio Molecular Dynamic Simulations at the Example of Two Transition Metal Catalyzed Reactions*, Int. J. Mol. Sci. **2011**, 12(2), 1389–1409

[1] P. J. di Dio, S. Zahn, C. B. W. Stark, B. Kirchner: *Understanding Selectivities in Ligand-free Oxidative Cyclizations of 1,5- and 1,6-Dienes with RuO_4 from Density Functional Theory*, Z. Naturforsch. **2010**, 65b, 367–375

Preprints

[P7] P. J. di Dio: *On the periodic Navier–Stokes equation: An elementary approach to existence and smoothness for all dimensions $n \geq 2$.*, arXiv:2010.05579

[P6] P. J. di Dio: *Transformations of Moment Functionals*, arXiv:2007.13347

[P5] P. J. di Dio: *Schwartz Function Valued Solutions of the Euler and the Navier–Stokes Equations*, arXiv:1912.11705

[P4] P. J. di Dio: *The multidimensional truncated Moment Problem: Shape and Gaussian Mixture Reconstruction from Derivatives of Moments*, arXiv:1903.00790

[P3] P. J. di Dio, M. Kummer: *The multidimensional truncated Moment Problem: Carathéodory Numbers from Hilbert Functions*, arXiv:1903.00598

[P2] P. J. di Dio, K. Schmüdgen: *The multidimensional truncated Moment Problem: The Moment Cone*, arXiv:1809.00584

[P1] P. J. di Dio: *Weyl Circles for one-dimensional Moment Problems*, arXiv:1506.06589

Peer Reviewer for Journals and Books

- Handbook of Numerical Analysis
- Mathematics
- SIAM Journal on Applied Algebra and Geometry
- Studies in Applied Mathematics

Grants and Scholarships

- PhD scholarship, 2014, Max Planck Institute for Mathematics in the Sciences, Leipzig
- PhD/research grant, 2016, Deutsche Forschungsgemeinschaft (DFG), SCHM1009/6-1, by Prof. Konrad Schmüdgen

Languages

- German - native
- English - fluent

References

Name	Prof. Dr. Konrad Schmüdgen
Affiliation	University of Leipzig, Germany
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Name	Prof. Dr. Mario Kummer
Affiliation	Technical University Dresden, Germany
Position	Junior-Professor
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Name	Prof. Dr. Tatjana Eisner
Affiliation	University of Leipzig, Germany
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