On the Shapiro polynomials and the PONS construction

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The standard discrete Walsh functions, used to perform the Hadamard transform, are one of the basic objects in discrete harmonic analysis, due to their efficiency in solving number of tasks in digital signal processing. The aim of this work was to investigate another powerful tool for signal processing, the less known orthogonal set of discrete functions PONS (Prometheus Orthonormal Set), also taking values only ± 1, and forming basis of arbitrary size.

PONS was originally constructed to prove an uncertainty principle conjecture of H. S. Shapiro, and it is based on the Shapiro sequences.

In order to investigate the essential property of PONS — symmetry, we refer to the Shapiro polynomials and their properties. We present explicit formula for the coefficients of the Shapiro polynomials of first and second kind, and derive several relations among them. The main aim of this work was to study the problem of the determination of Lebesgue constants of the PONS matrices, which are Hadamard, and the rows of which represent the previous mentioned discrete PONS functions. The properties of the corresponding Dirichlet kernel are investigated. Relationships are derived between two Dirichlet kernels, one of which is twice the size of the other. Interesting properties of the Lebesgue constants were obtained, among which is a formula for recursive calculation of an arbitrary Lebesgue constant.

References

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Keywords
PONS, Shapiro polynomials, Hadamard matrices, Uncertainty principle, Lebesgue constants, Dirichlet kernel