Alpha-Molecules (Abstract)

In order to efficiently represent multivariate data, which are often governed by anisotropic features – consider images with edges for example – many new representation systems beyond wavelets have been developed over the last decade. Since there is by now a whole zoo of such directional systems – ridgelets, curvelets and shearlets to name just a few – it is desirable to have some common framework, which builds upon their essential similarities. Such a framework would allow to deduce general results for many representation systems simultaneously.

In this talk we want to present the concept of α-*molecules*, which generalizes the recently introduced *parabolic molecules*. Systems of α-molecules feature a characteristic tiling of the frequency plane and an anisotropic scaling law, which is specified by the parameter α. Hence the concept incorporates the main features, which are common to most directional systems, and it is general enough to comprise the classical (radial) wavelet systems ($\alpha = 1$), ridgelets ($\alpha = 0$) as well as curvelets and shearlets ($\alpha = 1/2$).

As an application of the concept, we analyze the sparse approximation behavior of α-molecules. For this the notion of *sparsity equivalence* is introduced. With the help of this notion, it is possible to identify large classes of α-molecules providing the same sparse approximation results. In view of these results, one natural consequence is that curvelets and shearlets exhibit the same approximation behavior.

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