BOUNDARY EFFECTS, BOUNDARY LAYER POTENTIALS AND THE POLYHARMONIC DIRICHLET PROBLEM

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The presence of a boundary poses a number of challenges to kernel approximation. One well-understood challenge for surface spline approximation comes via a basic saturation result: on bounded domains, the highest attainable rate of approximation is far lower than that of the analogous boundary-free setting. A related problem is that for many standard function classes, precise rates of approximation have been unknown for surface spline approximation.

For fairly general domains in $\mathbb{R}^d$ (compact with smooth boundaries), we present an approximation scheme for surface spline approximation that delivers L_p, $(1 \leq p \leq \infty)$ approximation orders for well known smoothness spaces. Furthermore, for sets of centers having extra density near the boundary (violating a key assumption of the saturation result) the increased free-space convergence rates can be achieved. Connections between this scheme and boundary layer potential solutions of polyharmonic Dirichlet problems will be discussed.