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Nonuniform Sparse Recovery with Fusion Frames

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Fusion frames are generalization of classical frames that provide a richer description of signal spaces. Instead of frame vectors, we have a collection of subspaces $W_j \subset \mathbb{R}^d$, $j = 1, \ldots, N$, where d is the dimension of the ambient space. Then it holds that

$$A\|x\|_2^2 \leq \sum_{j=1}^{N} v_j^2 \|P_j x\|_2^2 \leq B\|x\|_2^2$$

for some universal fusion frame bounds $0 < A \leq B < \infty$ and for all $x \in \mathbb{R}^d$, where P_j denotes the orthogonal projection onto the subspace W_j. The generalization to fusion frames allows us to capture interactions between frame vectors to form specific subspaces that are not possible in classical frame theory. In this model, a sparse signal has energy in very few of the subspaces of the fusion frame, although it does not need to be sparse within each of the subspaces it occupies. This sparsity model is captured using a mixed ℓ_1/ℓ_2 norm for fusion frames.

Main idea is to extend the concepts of Compressed Sensing to fusion frames. As in classical compressive sensing setup, a sparse signal in a fusion frame can be sampled using very few random projections and exactly reconstructed using a convex optimization that minimizes the mixed ℓ_1/ℓ_2 norm. In this talk, we provide a nonuniform recovery result for fusion frames. In other words, we have a sparse fixed but unknown vector in our fusion frame and we seek conditions on the random (Rademacher or Gaussian) measurement matrix that guarantee the recovery of the sparse vector via convex minimization with high probability. Novelty in our research is to exploit the property of fusion frames which is the angle between frame subspaces. If λ denotes the cosine of minimum mutual angle between frame subspaces, then number of measurements m needs to be order of

$$\mathcal{O}((1 + \lambda s) \ln(N)$$

in order to recover s-sparse vector with high probability. As one can see, as λ becomes smaller, i.e., as frame subspaces get closer to being mutually orthogonal, number of measurements needed for recovery decreases. We will further give a preliminary result towards the uniform recovery for fusion frames that is similar to the nonuniform result.