Phase Retrieval from very few Measurements

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Phase Retrieval is a well-known problem from mathematical signal processing which arises in, e.g., X-ray crystallography, holography, and optical coherence tomography. The task is to recover signals (up to a global phase factor) from a set of intensity measurements, that is, the absolute values of linear measurements:

Definition: A system $\{v_k\}_{k=1}^M \subset \mathbb{C}^N$ *allows for Phase Retrieval* if for all $x, y \in \mathbb{C}^N$ we have

$$|\langle x, v_k\rangle| = |\langle y, v_k\rangle| \; \forall k = 1, \dots, M \quad \Longrightarrow \quad y = \lambda x, \; |\lambda| = 1.$$

In a recently published paper of Bandeira, Cahill, Mixon, and Nelson the following is conjectured:

(a) Systems with less than $4N-4$ vectors do not allow for Phase Retrieval.

(b) A generic system consisting of at least $4N-4$ vectors allows for Phase Retrieval.

In the talk, we provide a system consisting of $4N-4$ vectors and show by very simple means that it indeed allows for Phase Retrieval. Also a reconstruction algorithm will be presented.