

3D Parabolic Molecules

We live in a world which is constantly overflowed by data. For example, astronomers all over the world continuously collect data by photographing the night sky. It is of high importance that these images are saved in an efficient manner.

An image can be mathematically modelled by a cartoon-like function $f \in L^2(\mathbb{R}^2)$. In order to reduce the data needed to save the image represented by f , one tries to write f as a linear combinations of some functions $(\psi_i)_{i \in I}$ (which for instance could form a frame),

$$f = \sum_{i \in I} c_i \psi_i,$$

and then hope that most c_i 's are small - to hope that the coefficient sequence $(c_i)_{i \in I}$ is *sparse*. The small coefficients can be left out without making too large an error, and the amount of data to same f can be reduced.

It is now of large importance to choose the system $(\psi_i)_{i \in I}$ in such a way so that the sequences $(c_i)_{i \in I}$ become sparse - or at least have a fast decay. Two types of systems which have been proven succesful are *Curvelets* and *Shearlets*. They both base on the same idea - take one function ψ and then generate the systems of functions by parabolic scaling, rotating/shearing and translating:

$$\psi_{j,\theta,k}(x) = \psi(D_{2^j} R_\theta(x - k))$$

When ψ is chosen wisely, the systems can be proven to provide optimally sparse approximation rates for cartoon-like functions.

The similiarities of Curvelets and Shearlets are striking, and one it could be asked if it is not possible to bring them under one roof. In fact this is possible. The roof was designed by Grohs and Kutyniok in 2012 and it is called *Parabolic Molecules*. The main idea behind them is still to parabolically scale, rotate and translate a generator. The parameters are however allowed to be chosen more freely than in the Curvelet case, and it is also allowed for every function m_λ in the system to have its own generator $a^{(\lambda)}$.

Making some assumptions on the generators, one can prove that the Gramian matrix $(m_\lambda, p_\mu)_{\lambda,\mu}$ of any two parabolic molecules exhibit a strong off-diagonal decay. This property can be used to, essentially, prove that any tight frame which consists of a system of parabolic molecules provide almost optimal sparse approximation rates.

This talk will give a more thorough description of the concept as well as present a generalization to three dimensions, which is partly novel work.