Analysis of Inpainting via Hybrid Shearlets and Clustered Sparsity

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Abstract

Inpainting is generally understood to be the process of recovering corrupted data signals. Problems with damaged or missing data often occur in the field of imaging science; just imagine, for example, seismic data measured by an array of sensors with a more or less large number of them failing due to electronic defects. This will result in an image with a bunch of small missing stripes. In the past decades, various algorithms have been developed to solve the problem of (image-)inpainting. However, most of the respective approaches are of empirical nature, and a theoretical foundation is often missing.

To meet this challenge, the novel concept of *clustered sparsity* has been developed recently and turned out to be a useful tool in the mathematical analysis of inpainting. The fundamental idea underlying this method is based on the assumption that a given signal in Hilbert space $\mathcal{H}$ possesses some "significant structure", which can be "efficiently" represented by an appropriate Parseval frame Φ for $\mathcal{H}$.

In this talk, an abstract inpainting framework is introduced which provides two recovering algorithms as well as some suitable error estimates. Based on clustered sparsity, they are motivated by the ideas of *compressed sensing*. Concerning two dimensional images, these methods will be applied to a continuous model in $L^2(\mathbb{R}^2)$, which is governed by a line distribution. To capture this anisotropic feature, we will chose Φ to be a *shearlet system*. A novel construction of smooth Parseval frames of hybrid shearlets will be presented, which enables the treatment of shearlets and wavelets in an uniform manner. Provided that the gaps of the corrupted image are appropriately bounded, the main result finally shows that one can indeed achieve *asymptotically perfect inpainting*. In this context, it turns out that—compared to classical wavelets—shearlets require much lower assumptions on the gap size.

Keywords: Inpainting, Microlocal Analysis, Shearlets, Wavelets