Can everything be computed? - On the Solvability Complexity Index and Towers of Algorithms

Prof. Dr. Anders Hansen
Department of Applied Mathematics and Theoretical Physics
Centre for Mathematical Sciences,
University of Cambridge
a.hansen@damtp.cam.ac.uk

In this talk we will address some of the fundamental barriers in the theory of computations. This is done via the concept of the Solvability Complexity Index (SCI). The SCI is the smallest number of limits needed in order to compute a desired quantity (spectra of operators, roots of polynomials, solutions to linear equations etc.). In particular, several of the fundamental problems in computations have SCI greater than one (more than one limit is needed). This may come as a surprise, however, this touches onto the fundamental boundaries of computational mathematics. In several cases (spectral problems, inverse problems) one can provide sharp results on the SCI, thus establishing the absolute barriers for what can be achieved computationally. For example, it has recently been shown that the SCI of spectra of self-adjoint infinite matrices is equal to two. This result yields the first algorithm to compute such spectra in two limits. Moreover, we will discuss recent bounds for the SCI of spectra of classes of Schrödinger operators, thus one can affirmatively answer the long standing question on whether or not these spectra can actually be computed. Fortunately, these bounds are so low that the algorithms provided are not just theoretical, but indeed feasible. Finally, we will discuss how the SCI provides a natural framework for understanding barriers in computations, and in particular, how the fundamental barriers of Doyle and McMullen on solving the quintic by iterations can be put in the SCI framework.