Global registration of multiple point clouds using semidefinite programming

Prof. Dr. Amit Singer
Mathematics Department,
Princeton University
amits@math.princeton.edu

Consider N points in d-dimension and $M > 2$ local coordinate systems that are related by unknown rigid transforms. For each point we are given (possibly noisy) measurements of its local coordinates in some of the coordinate systems. Alternatively, for each coordinate system we observe the coordinates of a subset of the points. We study the problem of estimating the global coordinates (up to a global rigid transform) of the points from such measurements. This problem comes up in distributed approaches to molecular conformation and sensor network localization, astrometry, and also in computer vision and graphics applications. The least-squares formulation, though non-convex, has a well-known closed-form solution for the case $M = 2$ (based on the singular value decomposition). However, no closed form solution is known for $M > 2$. In this talk, we propose a semidefinite relaxation of the least-squares formulation, and prove conditions for exact and stable recovery for both this relaxation and for a previously proposed spectral relaxation. In particular, using results from rigidity theory and the theory of semidefinite programming, we prove that the semidefinite relaxation can guarantee recovery under more adversarial measurements compared to the spectral counterpart. In the second part of the talk, we prove a worst-case performance guarantee for the semidefinite program, in the form of a constant approximation ratio, which holds for several applications including the orthogonal Procrustes problem. The approximation ratio is given in terms of the expected average singular value of a random Gaussian matrix with iid entries. This approximation ratio improves over the best previously known approximation ratio of $1/(2\sqrt{2})$ proposed in the context of the non-commutative Grothendieck inequality.