Analysis of Generalized Ridge Functions in High Dimensions

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Abstract

The approximation of functions in many variables suffers from the so-called "curse of dimensionality". Namely, functions on $\mathbb{R}^N$ with smoothness of order s can be recovered at most with an accuracy of $n^{-s/N}$ applying n-dimensional spaces for linear or nonlinear approximation. However, there is a common belief that functions arising as solutions of real world problems have more structure than usual N-variate functions. This has led to the introduction of different models for those functions. We study generalized ridge functions of the form

$$\mathbb{R}^N \ni x \mapsto f(x) = g(\operatorname{dist}_\pm(x, M)),$$

where M is a d-dimensional, smooth submanifold of $\mathbb{R}^N$, $g \in C^s(\mathbb{R})$ and $\operatorname{dist}_\pm(\cdot, M)$ is the signed distance.