Shearlets for the simulation of natural phenomena

Philipp Petersen
Technische Universität Berlin

Shearlet systems form representation systems that behave optimally when used to represent certain functions appearing frequently in natural images. These functions exhibit sharp jumps along smooth curves and are coined cartoon-like functions. The optimal approximation properties of shearlet systems for these functions has been extensively employed in mathematical signal and image processing. However also in mathematical modeling, functions with similar characteristics play an important role. For instance in wave scattering applications, as for instance ultra sound or radar, inhomogeneities usually admit sharp changes in contrast.

Another instance of the appearance of functions that are smooth apart from one singularity curve in natural science appears when modeling fractures. In this talk I will present a model for the simulation of brittle fracture. Brittle fracture can be modeled by using an energy minimization approach that leads to the *Mumford-Shah model*. While solving the Mumford-Shah function is a hard problem, relaxations and approximations have been proposed, that are more accessible. One approximation is given by the *Ambrosio-Tortorelli functional* which will be the basis for our considerations.

I will propose a formulation in terms of shearlet coefficients that closely resembles the Ambrosio-Tortorelli functional. We will see some first numerical results as well as limitations of this approach. We will discuss how these limitations can be overcome by introducing a new class of *shearlets systems adapted to bounded domains*.