Gelfand numbers, their connection to Compressed Sensing and Carl's Inequality

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In its basic setting, compressed sensing studies pairs (A, Δ) of linear measurement maps $A \in \mathbb{R}^{m,d}$ and (non-linear) recovery maps $\Delta : \mathbb{R}^m \to \mathbb{R}^d$, such that the recovery error $x - \Delta(Ax)$ is small for k-sparse vectors

$$x \in \Sigma_k = \{x \in \mathbb{R}^d : \#\{i : x_i \neq 0\} \leq k\}.$$

To allow for stability needed in applications, it is also necessary that the methods of compressed sensing are extendable to compressible vectors $x \in K$ for some set $K \subset \mathbb{R}^d$ consisting of vectors which can be well approximated by sparse vectors.

In this talk we will see that the performance of an optimal pair (A, Δ) with respect to some set K is equivalent to the Gelfand number of K. Further, with the concept of entropy numbers and Carl's inequality, we will calculate Gelfand number for $K = \ell_p^d$.

Results presented in this talk are joint work with Aicke Hinrichs and Jan Vybíral.