Geometric properties of solutions to the total variation denoising problem

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To give some background, TV denoising has been extensively studied in imaging sciences since its introduction by Rudin, Osher and Fatemi in 1992. However, the majority of theoretical works on TV denoising are actually concerned with the regularized solutions in the absence of noise. For instance, it is known that the jump set of the regularized solutions are contained in the jump set of the original data, but this knowledge is fairly meaningless in the presence of L^2 noise since the noisy function can introduce arbitrarily many new discontinuities. Furthermore, works that consider the impact of noise on TV regularized solutions typically derive integral estimates such as L^2 error bounds or bounds on the total variation of the solution. However, such estimates do not inform on the proximity of the gradient support of the regularized solution to that of the clean function. In particular, they do not completely explain the ability of total variation denoising to remove undesired oscillations in the noisy data.

To fill this theoretical gap, I will address the following question: Given a small neighbourhood around the support of Df where f is the clean function, when will the gradient support of the regularized solution be contained inside this neighbourhood if the noise function has sufficiently small L^2 norm?