Image Approximation with α-Molecules

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The class $\mathcal{E}^\beta(\mathbb{R}^2)$, $\beta > 0$, of cartoon-like functions is an established standard model for image data. A cartoon-like function consists of two C^β-regions separated by a C^β-singularity curve, modeling an edge in an image. The observation that classical wavelet systems cannot optimally represent such functions led to the development of new multi-scale systems such as e.g. ridgelets, curvelets, and shearlets, which are better suited for this task.

Ridgelets have been shown to optimally resolve images from the subclass $E^\beta(\mathbb{R}^2) \subset \mathcal{E}^\beta(\mathbb{R}^2)$, where the edge curves are straight. It is also a classical result, that curvelets and shearlets provide optimal representation for the class $\mathcal{E}^2(\mathbb{R}^2)$. The decisive feature is the inherent scaling law of the systems: whereas wavelets scale isotropically, curvelets and shearlets scale parabolically, ridgelets only in one direction. This observation motivated the introduction of so-called α-curvelets (and also α-shearlets), where the parameter $\alpha \in [0, 1]$ specifies the scaling anisotropy. These systems constitute a family of representation systems stretching from wavelets ($\alpha = 1$) on the one end to ridgelets ($\alpha = 0$) on the other end, while the classical curvelets (or shearlets, respectively) are obtained for $\alpha = 1/2$.

In this talk we will investigate the sparse approximation performance of α-curvelets with respect to the classes $\mathcal{E}^\beta(\mathbb{R}^2)$ and $E^\beta(\mathbb{R}^2)$, with the main focus on the influence of the parameter $\alpha \in [0, 1]$. The results obtained for α-curvelets also have direct implications on other α-scaled systems, such as e.g. α-shearlets. Utilizing the framework of α-molecules, which is a unifying framework for α-scaled systems, it is possible to transfer these approximation results to other systems of α-molecules and identify large classes of systems providing the same sparse approximation behavior.