Stable reconstructions for the analysis formulation of ℓ^p-minimization using redundant systems

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Abstract

Sparse solutions are usually obtained in compressed sensing by solving a convex minimization problem of the form

$$\min_x \|x\|_1 \quad \text{subject to} \quad y = Ax, \tag{1}$$

where $A \in \mathbb{R}^{m\times n}$ is the *sensing matrix*, $y \in \mathbb{R}^m$ is the *measurement vector* and m is much smaller than n. However, real world signals are often not directly sparse but rather sparse in some transform domain. For that reason, the *analysis formulation* of (1) is often considered, that is

$$\min_x \|\Psi x\|_p^p \quad \text{subject to} \quad \|y - Ax\|_2 \leq \varepsilon, \tag{2}$$

with Ψ being the analysis operator of an arbitrary redundant frame and $\varepsilon > 0$ controlling the noise level.

We discuss the existence and the stability of such solutions. Moreover, we draw a comparison between ℓ^p-mininimization and ℓ^1-minimization by investigating the constants of the error bounds. We will also provide an algorithm that solves an approximate problem and discuss its convergence. Finally, we provide numerical experiments for the wavelet transform and the shearlet transform in (2) that show the potential of ℓ^p-minimization over standard ℓ^1-minimization in the context of Fourier measurements.

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