Compressed Sensing with Neural Networks

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In compressed sensing, one has a vector of measurements $y \in \mathbb{R}^m$, which is gained from a signal $x^* \in \mathbb{R}^n$ via noisy linear measurements

$$y = Ax^* + \eta.$$

As one wants to keep the number of measurements m low, the equation system $Ax = y$ is highly underdetermined. To ensure that x^* is the unique solution in S of the equation system, one assumes to have a certain prior knowledge on x^*, i.e.

$$x^* \in S$$

for a set S. Let's call this property recoverability. Usually S is sparsity-based, e. g. S consists of k-sparse vectors in $\mathbb{R}^n$ for a certain k. A new approach is to use neural networks as generators, i.e. choose

$$S := \operatorname{ran}(G)$$

for a network $G : \mathbb{R}^k \to \mathbb{R}^n$. [1] showed recoverability under mild conditions. This talk deals with the question, whether and where this approach gives advantages or bears weaknesses in commparison to the established sparsity-based approaches. For example, numerical experiments indicate that a network-based model "fits real world data better" than sparsity-based. On the other hand, even though one has recoverability, finding x^* requires solving a nonconvex optimization problem. It can be shown that there are instances of this problem, which are not solvable in polynomial time (unless $P = NP$).

References

[1] A. Bora, A. Jalal, E. Price, and A. G. Dimakis. Compressed sensing using generative models. In *Proceedings of the 34th International Conference on Machine Learning-Volume 70*, pages 537–546. JMLR. org, 2017.