

Imaging Science Meets Compressed Sensing

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Abstract:

Modern imaging data are often composed of several geometrically distinct constituents. For instance, neurobiological images could consist of a superposition of spines (pointlike objects) and dendrites (curvelike objects) of a neuron. A neurobiologist might then seek to extract both components to analyze their structure separately for the study of Alzheimer specific characteristics. However, this task seems impossible, since there are two unknowns for every datum.

Compressed sensing is a novel research area, which was introduced in 2006, and since then has already become a key concept in various areas of applied mathematics, computer science, and electrical engineering. It surprisingly predicts that high-dimensional signals, which allow a sparse representation by a suitable basis or, more generally, a frame, can be recovered from what was previously considered highly incomplete linear measurements, by using efficient algorithms.

Utilizing the methodology of Compressed Sensing, the geometric separation problem can indeed be solved both numerically and theoretically. For the separation of point- and curvelike objects, we choose a deliberately overcomplete representation system made of wavelets (suited to pointlike structures) and shearlets (suited to curvelike structures).

A classical decomposition principle is to minimize the ℓ_1 norm of the representation coefficients. For this, our theoretical results, which are based on microlocal analysis considerations, show that at all sufficiently fine scales, nearly-perfect separation is indeed achieved. A second possible decomposition principle is iterative thresholding. Surprisingly, in this situation we can prove even stronger theoretical results, namely the perfect separation of the wavefront sets of the respective components.

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