The Framework of α-Molecules:
Theory and Applications

Martin Schäfer
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For the efficient representation of multivariate data, which is often governed by anisotropic features – consider images with edges for example – many new representation systems beyond wavelets have been developed over the last decade. Some prominent examples among those are ridgelets, curvelets, and shearlets, to name a few.

Conceived as a unifying concept, the notion of α-molecules builds upon the essential similarities of such systems. A characteristic property is the employed scaling law, specified by the parameter $\alpha \in [0, 1]$, and the time-frequency localization of the system elements. Graphically, this can conveniently be pictured by the induced tiling of the frequency domain.

Particular examples of α-molecules are wavelets for $\alpha = 1$, ridgelets for $\alpha = 0$, further curvelets and shearlets for $\alpha = 1/2$. But also more general directional systems are included. The advantage of the common framework is that it allows to investigate properties of those systems simultaneously.

In this talk we will present the definition of α-molecules, some of their essential properties, as well as applications in approximation theory and coorbit theory.