Learning derivatives with deep ReLU neural networks: A trade-off between regularity and complexity

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Abstract

Powered by today's improved computational resources and the immense amount of accessible data, deep neural networks substantially outperform both traditional modelling approaches, e.g. based on differential equations, and classical machine learning methods in a wide range of applications. Prominent areas of application include image processing, speech recognition, and natural language processing.

Viewing learning algorithms as function approximators, their ability to express a rich class of functions, called expressiveness, is a necessary condition for disentangling complicated feature dependencies and to perform well in challenging real-world applications.

In some cases, it is possible and can be useful to include information about the derivates of the function to be approximated in the learning process. This holds true, for example, in network compression tasks or for numerically solving partial differential equations with neural networks.

In this talk, we study the expressiveness of neural networks. We construct deep ReLU neural networks that approximate functions from Sobolev spaces and their (fractional) gradient and derive upper bounds for the complexity of the network. We establish a trade-off between the regularity used in the approximation norm and the complexity that is at most needed for the network.

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