An Introduction into Fourier Analysis on Graphs in the Light of Deep Learning

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Abstract

We shall give an introduction into the theory of *graph-functions*, i.e., maps from the vertex set of a graph into $\mathbb{R}$. In particular, we want to explain the crucial role of the *graph-Laplacian* operator, which can be used to define smoothness for such functions. Moreover, a Fourier theory can be set up by considering the eigendecomposition of the graph-Laplacian.

We will then take a look at two different representation systems for graph functions: on the one hand wavelets defined via spectral graph theory and on the other hand a parametric dictionary, whose parameters are learned. The atoms of these systems are translations of a kernel function, defined in the spectral domain, to different vertices of the graph.

Finally, we address the representability of such functions by neural networks. Here, a big issue is the encoding of the vertex set by real numbers such that the vertices are tractable for a neural network. We will show various ideas of assigning $\mathbb{R}^d$-labels to the vertices. Nevertheless, a graph does not posses a Euclidean structure in general, which is why each approach has its downsides.