Nichtlineare Dynamik und deren Anwendungen. Eine Einführung.
TU Berlin, Institute of Mathematics. Prof. S. Yanchuk
Test Questions.

1. Definition of dynamical system. Examples of dynamical systems (continuous vs. discrete time; continuous vs. discrete phase space). Reduction of ODEs to a system of the 1st order. Classification: autonomous vs. nonautonomous; linear vs. nonlinear. Illustrate the above using the example of a damped harmonic oscillator.

2. One-dimensional flows: Equilibria and their stability (geometric and analytic approach), linearization.

3. Saddle-node bifurcation and transcritical bifurcation for 1D-flows, normal form (without derivation), bifurcation diagrams.

4. Pitchfork bifurcations for 1D flows, normal form (without derivation), bifurcation diagrams, sub- versus supercritical bifurcation.

5. Describe how the pitchfork bifurcation is perturbed in the system $\dot{x} = h + rx + x^3$. Cusp point. What is 2D bifurcation diagram?

6. Flows on the circle: uniform oscillator. Explain qualitatively the dynamics and bifurcations of non-uniform oscillator (ghost and bottleneck, homoclinic orbit). How the period of a non-uniform oscillator can be found?

7. "Fireflies example" - periodically perturbed phase oscillator, frequency locking.

8. 2D linear flows and their classification (you are allowed to take the bifurcation diagram with respect to τ and Δ parameters on the exam).

9. Different stability definitions and examples: Attracting, globally attracting, Lyapunov stable, neutrally stable, asymptotically stable, unstable.

10. 2D flows. Linearization at equilibrium. Hyperbolic equilibria (fixed points), structural stability.

11. Conservative systems: definition, impossibility of attracting equilibria. Derive energy function for the system $m\ddot{x} = F(x)$.

12. Phase portrait for the system $m\ddot{x} = F(x)$ in a double-well potential, i.e. $F(x) = -\frac{dU}{dx}$ with $U(x) = -\frac{1}{2}x^2 + \frac{1}{4}x^4$. Using this example, explain how centers are perturbed in conservative and non-conservative systems.

13. Reversible systems: definition and geometric properties.

14. Index of a closed curve: definition and main properties.

15. Limit cycle: definition, example $\dot{r} = r(1 - r^2)$, $\dot{\theta} = 1$.