Research Center MATHEON
Mathematics for Key Technologies

MATHEON Multiscale Seminar*

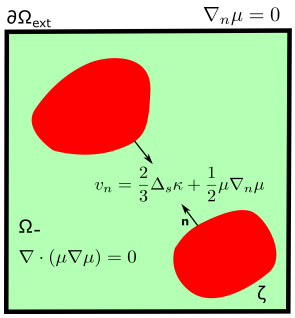
organised by R. Klein (FU), K. Schmidt (TU), B. Wagner (TU), and S. Nesenenko (U Duisb.-Essen)

TU Berlin, MA 313, **Wednesday, July 15th, 2015**, 9.15 a.m.

Andreas Münch (University of Oxford, 9.15 a.m.)

Asymptotic analysis of phase-field models involving surface diffusion

Phase field models frequently provide insights to phase transitions, and are robust numerical tools to solve free boundary problems corresponding to the motion of interfaces. A body of prior literature suggests that interface motion via surface diffusion is the long-time, sharp interface limit of microscopic phase field models such as the Cahn-Hilliard equation with a degenerate mobility function. Contrary to this conventional wisdom, we show via a careful asymptotic analysis involving the matching of exponential terms that the the long-time behaviour of a degenerate Cahn-Hilliard equation with a polynomial free energy leads to a sharp interface model that couples bulk and surface diffusion, thus permitting coarsening.



— Coffee break —

Patrik Marschalik (Universität Mainz, 10.35 a.m.)

Fundamental concepts of the methods of matched asymptotic approximations and multiple scales expansions

There are mainly two distinct approaches to construct approximate solutions of multiple scale problems: First the method of matched asymptotic approximations and second the method of multiple scale expansions.
The first approach seeks for approximations which are uniformly valid in certain subdomains and then matches these approximations to a composite approximation which is uniformly valid on the whole domain. There are two main matching techniques. First the intermediate matching based on extension theorems and the overlap hypothesis, and secondly the matching based on Van Dyke's asymptotic matching principles. Wiktor Eckhaus' elegant notation allows us to consistently formulate these matching principles.
For the second approach several new independent variables are introduced. In this way, the original problem is embedded in a higher-dimensional space of independent variables, the new coordinates representing asymptotically rescaled spacio-temporal dependencies. In the following process one finds an expansion that is not uniformly valid on the whole domain per se. In order to obtain uniform validity one needs a rule, the non-secularity condition, that provides an appropriate guide in eliminating certain indeterminacies in the expansions so that uniform validity is achieved.
In this talk the concepts behind these two approaches are discussed. We will see how the two matching techniques interrelate and that one has to be very careful in stating the non-secularity condition when utilizing multiples scales expansions. Finally we will discuss differences between the two approaches. The focus of the talk is on the concepts rather than applications.

The methods to be discussed are ideally suited for studying, e.g., the detailed structure and motion of slender aircraft trailing wings. (Picture source: http://eaa650.blogspot.de)

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* The MATHEON Multiscale Seminar takes place one to two times per term with two talks about recent work on partial differential equations with multiple scales. Please contact one of the organisers if want to be invited by e-mail or if you would like to contribute a talk.
Webpage: `http://www.tu-berlin.de/?multiscale-seminar`