Research Center MATHEON
Mathematics for Key Technologies

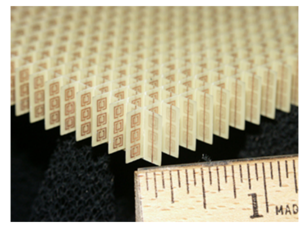
MATHEON Multiscale Seminar*

organised by R. Klein (FU), K. Schmidt (TU), B. Wagner (TU), and S. Nesenenko (U Duisb.-Essen)

TU Berlin, MA 313, **Friday, October 30th, 2015**, 9.15 a.m.

Agnes Lamarcz (Universität Dortmund, 9.15 a.m.)

Effective Maxwell's equations in a geometry with flat split-rings



source: Wikipedia

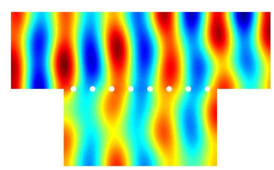
Propagation of light in heterogeneous media is a complex subject of research. It has received renewed interest in recent years, since technical progress demands smaller devices and offers new possibilities. At the same time, theoretical ideas inspired further research. Key research areas are photonic crystals, negative index metamaterials, perfect imaging, and cloaking.

The mathematical analysis of negative index materials, which we want to focus on in this talk, is connected to a study of singular limits in Maxwell's equations. We present a result on homogenization of the time harmonic Maxwell's equations in a complex geometry. The homogenization process is performed in the case that many (order η^{-3}), small (order η^1), thin (order η^2) and highly conductive (order η^{-3}) metallic split-rings are distributed in a domain $\Omega \subset \mathbb{R}^3$. We determine the effective behavior of this metamaterial in the limit $\eta \searrow 0$. For $\eta > 0$, each single conductor occupies a simply connected domain, but the conductor closes to a ring in the limit $\eta \searrow 0$. This change of topology allows for an extra dimension in the solution space of the corresponding cell-problem. Even though both original materials (metal and void) have the same positive magnetic permeability $\mu_0 > 0$, we show that the effective Maxwell system exhibits, depending on the frequency, a negative magnetic response. Furthermore, we demonstrate that combining the split-ring array with thin, highly conducting wires can effectively provide a negative index metamaterial.

— Coffee break —

Adrien Semin (TU Berlin, 10.35 a.m.)

When a thin periodic layer meets corners: asymptotic analysis of a singular Helmholtz problem.



Perforated liner inside an acoustic wave-guide.

We study here the acoustic wave propagation in a bounded domain made of a thin and periodic layer of finite length placed into an medium that admits periodic inhomogeneities in a vicinity of the layer. The difficulty of this problem is the presence of the re-entrant corners at both ends of the layer. We are interested in the construction and the analysis of the asymptotic expansion in the whole domain, using different ansatz in the far field region (away from the periodic layer), close to the periodic layer and away from its corners, and in the near field region (close to the corners). The interaction of the corner singularities and the periodic layer the asymptotic expansion shows up in powers of δ and δ^α, where δ is the layer thickness and periodicity of the inhomogeneities and α depends on the opening angle.

In the talk we will pay particular attention to analyze the near-field problems close to the corner and the periodic layer. For this, we will introduce a solution extending the Kondratiev theory for corner singularities (which is based on the Mellin transform) in the spirit of Nazarov. This will allow us give show rigorous estimates of the error of the asymptotic expansion. The talk will be finished by a series of numerical experiments illustrating the theoretical results on the example of perforated liners.

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* The MATHEON Multiscale Seminar takes place one to two times per term with two talks about recent work on partial differential equations with multiple scales. Please contact one of the organisers if want to be invited by e-mail or if you would like to contribute a talk.

Webpage: `http://www.tu-berlin.de/?multiscale-seminar`