Research Center MATHEON
Mathematics for Key Technologies

MATHEON Multiscale Seminar*

organised by R. Klein (FU), K. Schmidt (TU), B. Wagner (TU), and S. Nesenenko (TU)

TU Berlin, MA 313, **Wednesday, February 17th, 2016**, 14:00

Nicola Popovic (U Edinburgh, 14:00)

A geometric analysis of fast-slow models for stochastic gene expression

Stochastic models for gene expression frequently exhibit dynamics on different time-scales.One potential scale separation is due to significant differences in the lifetimes of mRNA and the protein it synthesises, which allows for the application of perturbation techniques [1, 2]. Here [3], we develop a dynamical systems framework for the analysis of a family of ”fast-slow” models for gene expression that is based on geometric singular perturbation theory [4]. We illustrate our approach by giving a complete characterisation of a standard two-stage model which assumes transcription, translation, and degradation to be first-order reactions. In particular, we develop a systematic expansion procedure for the resulting propagator probabilities that can in principle be taken to any order in the perturbation parameter. We verify our asymptotics by numerical simulation, and we explore its practical applicability, as well as the effects of a variation in the system parameters and the scale separation. Finally, we discuss the generalisation of our geometric framework to models for regulated gene expression that involve additional stages, which is a subject of ongoing research.

[1] V. Shahrezaei & P.S. Swain, Analytical distributions for stochastic gene expression, *Proc. Natl. Acad. Sci USA* 105, 2008.

[2] P. Bokes, J.R. King, A.T.A. Wood, & M. Loose, Multiscale stochastic modelling of gene expression, *J. Math. Biol.* 65, 2012.

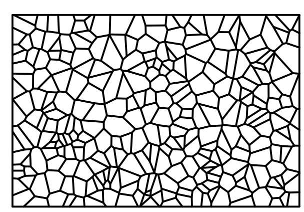
[3] N. Popovic, C. Marr, & P.S. Swain, A geometric analysis of fast-slow models for stochastic gene expression, *J. Math. Biol.* 72, 2016.

[4] C. Jones, Geometric singular perturbation theory, in Dynamical Systems, Lecture Notes in Math., 1995.

— Coffee break —

Martin Heida (WIAS Berlin, 15:20)

On Homogenization of Rate-independent Systems



A Voronoi-Tessellation as a stochastic model for a crystalline structure.

We study stochastic homogenization problems of the form

$$0 \in \partial\Psi_\varepsilon(\partial_t u^\varepsilon) + D\mathscr{E}_\varepsilon(t, u^\varepsilon), \qquad (1)$$

where $\mathscr{E}_\varepsilon : [0,T] \times B_\varepsilon \to \overline{\mathbb{R}}$ is a proper, quadratic functional and $\Psi_\varepsilon : B_\varepsilon \to \overline{\mathbb{R}}$ is proper and 1-homogeneous and B_ε is an ε-dependent Banach space. As usual in homogenization, the index $\varepsilon > 0$ is a smallnes parameter and (in general) relates to the scale of the underlying geometry of the physical system, such as crystaline structure, microscopic cracks etc.. We focus on Prandtl-Reuss plasticity and on elasticity problems coupled with Coulomb-friction.

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* The MATHEON Multiscale Seminar takes place one to two times per term with two talks about recent work on partial differential equations with multiple scales. Please contact one of the organisers if you want to be invited by e-mail or if you would like to contribute a talk.

Webpage: `http:://www.tu-berlin.de/?multiscale-seminar`