

# ASYMPTOTIC ANALYSIS

## Series 2

1. Given the following perturbed problem

$$\begin{cases} (u^\varepsilon)''(x) - (u^\varepsilon)'(x) + \varepsilon u^\varepsilon(x) = 0, & x > 0, \\ u^\varepsilon(0) = 0 \\ (u^\varepsilon)'(0) = 1 \end{cases} \quad (1)$$

Fix an asymptotic sequence  $\{\Phi_n(\varepsilon)\}_{n=0}^\infty$  and derive the asymptotic expansion of the solution  $u^\varepsilon$  of (1), without explicit computation of  $u^\varepsilon$ . Hint: what is the problem solved by the  $n$ -th term of the expansion?

2. Given the function

$$f(z) := \sinh(z^2)$$

for  $z \in \mathbb{C}$ . In which subdomain  $A \subset \mathbb{C}$  is  $f(z) \sim \exp(z^2)/2$  for  $|z| \rightarrow \infty$ ,  $\arg(z)$  fixed, and in which subdomain is  $f(z) \sim \exp(-z^2)/2$  for  $|z| \rightarrow \infty$ ,  $\arg(z)$  fixed?

3. With the aid of integration by parts or integration of the terms of an expansion of the integrand, compute the asymptotic expansion of

$$I(x) := \int_0^\infty \frac{\exp(-t)}{1+xt} dt \quad \text{for } x \rightarrow 0^+$$

Compute (numerically) the expansion for  $N$  terms, with  $N = 3, 4, 5, \dots, 26$  and plot the results. For comparison, integrate  $I(0.1)$  numerically, *e.g.* using the trapezoidal rule.

**To be handed in by:** May 26th, 2016 (2.15 pm, before lecture starts)

**Website:** <http://www.tu-berlin.de/\?asymptotic-analysis>

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