Technische Universität Berlin – Institut für Mathematik
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ASYMPTOTIC ANALYSIS

Series 3

1. Given the singular perturbed problem

$$\varepsilon x^2 - 2x + 1 = 0$$

use the iterative method to give the three first terms of the expansion of the two solutions of this problem, as $\varepsilon \to 0$.

2. Calculate the asymptotic expansions for following integrals. Use integration of the terms of an expansion of the integrand, integration by parts or the Gamma function.

a) $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ for $x \to 0^+$.

b) $\int_x^\infty e^{-t^4} dt$ for $x \to \infty$. Hint: use an integration by parts after rewriting $e^{-t^4} = f(t)\partial_t e^{-t^4}$

c) $\int_x^\infty e^{-t^4} dt$ for $x \to 0^+$. Hint: use that $\int_0^\infty e^{-t^\lambda} dt = \Gamma(1 + \frac{1}{\lambda})$

d) $\int_x^\infty e^{\mathrm{i}t} t^{-a} dt$, $a > 0$ for $x \to \infty$. Hint: use an integration by parts.

e) $\int_u^\infty \cos(\theta^2) d\theta$ for $u \to \infty$. Hint: use the integral computed just above.

3. Show that

$$\int_0^\pi t^x \sin(t) dt \sim \frac{\pi^{x+2}}{x^2}, \quad x \to \infty$$

Hint: use a change of variable to apply Watson's lemma.

To be handed in by: June 9th, 2016 (2.15 pm, before lecture starts)
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