Technische Universität Berlin – Institut für Mathematik
Dr. Kersten Schmidt

SoSe 2016

ASYMPTOTIC ANALYSIS

Series 5

1. Using the method of the steepest descend, show that

$$\int_0^1 e^{\mathrm{i}xt^2}dt \sim \frac{1}{2}\sqrt{\frac{\pi}{2x}}(1+\mathrm{i}) - \frac{\sqrt{\pi}}{2}e^{\mathrm{i}x}\sum_{n=0}^{\infty}\frac{\mathrm{i}^{n+1}}{\Gamma\left(\frac{1}{2}-n\right)x^{n+1}}, \quad x\to\infty$$

2. Let us consider the second order system

$$\begin{cases} \varepsilon u_\varepsilon''(x) + \varepsilon u_\varepsilon'(x) - u_\varepsilon(x) = \cos x, & x\in(0,\pi) \\ u_\varepsilon(0) = u_\varepsilon(\pi) = 0 \end{cases}$$

- Show that solution of this system has two boundary layers, one at each ends. Hint: what is the boundary conditions satisfied by the limit problem?
- We consider only the left boundary. Derive as asymptotic expansion of the solution using a *Poincaré type* expansion for both slow and fast variable. Is the expansion well-posed?
- Compute the exact solution of the homogeneous equation without boundary conditions, and look at the characteristic scale of the solutions. Then, derive a suitable asymptotic expansion of the solution.

3. Let us consider the second order system

$$\begin{cases} u_\varepsilon''(x) + u_\varepsilon(x) = 0, & x\in(-\varepsilon,\pi/2) \\ u_\varepsilon(-\varepsilon) = 0, \quad u_\varepsilon(\pi/2) = 1 \end{cases}$$

Note that the domain $\Omega(\varepsilon) = (-\varepsilon,\pi/2)$ is ε-dependent. For this problem, we will write a matched asymptotic expansion, considering the far-field domain $\Omega_{\text{far}}(\varepsilon) = (\varphi_-(\varepsilon),\pi/2)$ and the near-field domain $\Omega_{\text{near}}(\varepsilon) = (-\varepsilon,\varphi_+(\varepsilon))$.

- On the far-field domain, using a *Poincaré-type* expansion, write the problem solved by u_n. Then, give the general form of u_n.

See next page!

- On the near-field domain, using the change of variable $X = x/\varepsilon - 1$, write the problem solved by U_n. Show then that $U_{2n}(X)$ and $U_{2n+1}(X)$ are both polynomials of degree $2n + 1$.
- Write the matching conditions and link behaviour between (u_n) and (U_n).
- Justify the asymptotic expansion.
- What is the boundary condition satisfied at the near-field term at $x = 0$?
- What happens if the domain is $(-\varepsilon, \pi)$ instead?

To be handed in by: July 14th, 2016 (2.15 pm, before lecture starts)

This exercise series will be discussed in the tutorial class on July 20th, 2016, 2.15 p.m. in MA 376.

Website: `http://www.tu-berlin.de/?asymptotic-analysis1`

Coordinator: Vorlesung:
Dr. Kersten Schmidt, MA 363, `kersten.schmidt@math.tu-berlin.de`
Übung:
Dr. Anastasia Thöns-Zueva, MA 365, `anastasia.thoens@math.tu-berlin.de`