Technische Universität Berlin – Institut für Mathematik SoSe 2012
Dr. Kersten Schmidt

FEM for Electromagnetics and Wave Propagation

Series 1

1. Coloumb's law in $\mathbb{R}^2$
Assume that the world would be 2D and Gauß law

$$\operatorname{div} \mathbf{E}(\mathbf{x}) = -\Delta\varphi = \frac{1}{\epsilon_0}\varrho(\mathbf{x}) \tag{1}$$

would hold. How would be Coloumb's law?
Hint: Consider the Green's function $G(\mathbf{x}, \mathbf{x}') = G(|\mathbf{x}-\mathbf{x}'|)$ such that $\Delta_{\mathbf{x}} G(|\mathbf{x}-\mathbf{x}'|) = \delta(\mathbf{x}-\mathbf{x}')$ in polar coordinates where Δ is the Laplace operator and assume $\varrho(\mathbf{x})$ to be a point charge. Note, that

$$\int_V \delta(\mathbf{x}-\mathbf{x}')\mathrm{d}\mathbf{x} = \begin{cases} 1 & \text{if } \mathbf{x}' \text{ is in } V, \\ 0 & \text{otherwise.} \end{cases}$$

2. Electrostatic potential at infinity
Let $\varrho(\mathbf{x})$ be supported in some bounded area. How is the functional behaviour of $\varphi(\mathbf{x})$ and $\mathbf{E}(\mathbf{x})$ for $|\mathbf{x}| \to \infty$ in $\mathbb{R}^3$ (consider the asymptotically dominating term)?
How is the behaviour in $\mathbb{R}^2$ if (1) holds anyway.
Hint: Consider φ in spherical coordinates outside a ball enclosing the support of ϱ and use Fourier expansion in the angle directions.

3. Mesh generation in Matlab

a) Download the triangle mesh generator "mesh2d" (v24) from
`www.mathworks.com/matlabcentral/fileexchange/?term=mesh2d`
to some local directory.

b) Install the m-files by introducing the directory of the mesh generator to Matlab via `addpath()`.

c) Write a function `meshPlates.m` which creates a mesh for a circle with two parallel thin plates (rectanges) inside. The diameter, the distance between the plates, their width and the maximal mesh width are input parameters.

See next page!

d) (*advanced*) The task is like in (c), but with two thin plates of zero thinkness, i.e., two lines. Note, that there should be edges on both "sides" of the line, but one vertex at each end.
Hint: Take a thin trapezoidal and correct after the meshing the coordinates to be all on the line.

To be handed in by: April 25th, 2012 (4.15 pm, before lecture starts)
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