Technische Universität Berlin – Institut für Mathematik SoSe 2012
Dr. Kersten Schmidt

FEM for Electromagnetics and Wave Propagation

Series 2

1. Assume that the geometry and the sources are rotational-symmetric w.r.t. to the $\mathbf{e}_3$-axis. Write the electric formulation of the time-harmonic Maxwell's equations when searching the electric field possessing the same symmetry (no angle dependance in cylindrical coordinates). Use appropiate "rotation" operators $\mathrm{curl}_{\mathrm{cyl}}$ and $\mathbf{curl}_{\mathrm{cyl}}$ to decouple the three components of electric field.

2. Take the asymptotic behaviour of the electrostatic potential $\varphi(\mathbf{x})$ for $|\mathbf{x}| \to \infty$ for electric charges in a bounded area (see Exercise 2 in Series 1) to formulate an approximate impedance boundary condition at a spherical / radial boundary $r = R$

$$\nabla\varphi(\mathbf{x}) \cdot \mathbf{n} = \partial_r\varphi(\mathbf{x}) = -\beta(R)\varphi(\mathbf{x}).$$

What is $\beta(R)$ in $\mathbb{R}^2$ and $\mathbb{R}^3$?

3. Give an example of a function $u \in C^0(\Omega)$ for $\Omega = (0, 1)$

a) that does not possess a weak second derivative in Ω, or

b) that does not possess a weak third derivative in Ω, or

c) that does not possess a weak $\operatorname{div}\sigma\operatorname{grad}$ in Ω where you choose $\sigma(x)$ appropriately, but not constant.

d) Which conditions have to be added to the PDE

$$-\operatorname{div}\sigma\operatorname{grad} v + cv = f \text{ in } (0, \tfrac{1}{2}) \cup (\tfrac{1}{2}, 1)$$

to be equivalent to

$$-\operatorname{div}\sigma\operatorname{grad} v + cv = f \text{ in } \Omega,$$

where all differential operators are meant in weak sense.

See next page!

4. Provide a definition of the "weak" rotation $\mathbf{curl}\,\mathbf{x} \in (L^2(\Omega))^3$.
Hint: Use the integration by parts formula for **curl** given in task 6.

5. Consider the time-dependent Maxwell's equations in $\mathbb{R}^3$ without any assumption on the material laws, *i. e.*, take the original form with all fields $\mathbf{E}$, $\mathbf{D}$, $\mathbf{H}$, $\mathbf{B}$, $\mathbf{j}$ and ϱ, where all differential operators are meant in its weak sense. Let classical derivatives of sufficient order exist in non-overlapping bounded subdomains covering $\mathbb{R}^3$.
What are the continuity conditions for $\mathbf{E}$, $\mathbf{D}$, $\mathbf{H}$, $\mathbf{B}$, and $\mathbf{j}$ on the interfaces between subdomains?

6. (Voluntary) Show that for $\mathbf{f} \in (C^1(\Omega))^3 \cap (C^0(\overline{\Omega}))^3$

$$\int_\Omega \mathbf{curl}\,\mathbf{f}\,\mathrm{d}\mathbf{x} = \int_\Gamma \mathbf{n} \times \mathbf{f}\,\mathrm{d}S(\mathbf{x})$$

Hint: Use

$$\int_\Omega \mathbf{curl}\,\mathbf{u} \cdot \mathbf{f} - \mathbf{u} \cdot \mathbf{curl}\,\mathbf{f} \mathrm{d}\mathbf{x} = \int_\Gamma (\mathbf{u} \times \mathbf{f}) \cdot \mathbf{n} \mathrm{d}S(\mathbf{x})$$

for $\mathbf{u} = \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ the canonical basis of $\mathbb{R}^3$.

To be handed in by: May 8th, 2012 (1pm–2pm)
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