1. Let $V$ be a reflexive Banach space. Let $b(\cdot, \cdot) : V \times V \to \mathbb{R}$ be a bounded bilinear form that is $V$-elliptic. Show that $b$ fulfills the inf-sup conditions

$$\exists \gamma > 0 : \inf_{w \in U \setminus \{0\}} \sup_{v \in V \setminus \{0\}} \frac{|b(w, v)|}{\|v\|_V \|w\|_U} \geq \gamma,$$

(IS1)

$$\forall v \in V \setminus \{0\} : \sup_{w \in U \setminus \{0\}} |b(w, v)| > 0,$$

(IS2)

with $U = V$.

2. Consider the linear variational problem: find $u \in U$ such that

$$b(u, v) = \ell(v) \quad \forall v \in V.$$

(LVP)

Show the following statements:

a) If (IS1) does not hold then the solution $u$ of the linear variational problem (LVP) is not unique.

b) If (IS2) does not hold then a solution of the linear variational problem (LVP) does not necessarily exist.

Be prepared to present your results! Tutorial class on May 6th, 2014, 10.15am, MA 545

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