One- and Two-level Asynchronous Optimized Schwarz Methods for the solution of PDEs

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Asynchronous methods refer to parallel iterative procedures where each process performs its task without waiting for other processes to be completed, i.e., with whatever information it has locally available and with no synchronizations with other processes. For the numerical solution of a general partial differential equation on a domain, Schwarz iterative methods use a decomposition of the domain into two or more (possibly overlapping) subdomains. In essence one is introducing new artificial boundary conditions on the interfaces between these subdomains. In the classical formulation, these artificial boundary conditions are of Dirichlet type. Given an initial approximation, the method progresses by solving for the PDE restricted to each subdomain using as boundary data on the artificial interfaces the values of the solution on the neighboring subdomain from the previous step. This procedure is inherently parallel, since the (approximate) solutions on each subdomain can be performed by a different processor. In the case of optimized Schwarz, the boundary conditions on the artificial interfaces are of Robin or mixed type. In this way one can optimize the Robin parameter(s) and obtain a very fast method.

Instead of using this method as a preconditioner, we use it as a solver, thus avoiding the pitfall of synchronization required by the inner products. In this talk, an asynchronous version of the optimized Schwarz method is presented for the solution of differential equations on a parallel computational environment. A coarse grid correction is added and one obtains a scalable method. Several theorems show convergence for particular situations.

Numerical results are presented on large three-dimensional problems illustrating the efficiency of the proposed asynchronous parallel implementation of the method. The main application shown is the calculation of the gravitational potential in the area around the Chicxulub crater, in Yucatan, where an asteroid is believed to have landed 66 million years ago contributing to the extinction of the dinosaurs.